Problem 5.26

5.25 through 5.28 Determine by direct integration the centroid of the area shown.

\[ y = b \left(1 - \frac{k}{a}x^2\right) \]

For \( x = a, y = 0 \)

\[ c = b \left(1 - \frac{k}{a^2}\right) \]

\[ r = \frac{1}{a^3} \]

\[ y = b \left(1 - \frac{x^3}{a^3}\right) \]

\[ \bar{x} = \frac{1}{A} \int_0^a x \, dA = \frac{1}{A} \int_0^a y \, dx \]

\[ A = \int_0^a \left[b \left(1 - \frac{x^3}{a^3}\right)\right] \, dx = b \left[ x - \frac{x^4}{4a^3} \right]_0^a = \frac{3}{4} ab \]

\[ \bar{x} = \frac{1}{b} \int_0^a x \, dy = \frac{1}{b} \int_0^a y \, dx = \frac{1}{b} \int_0^a \left[ b \left(1 - \frac{x^3}{a^3}\right)\right] \, dx = \frac{1}{2} \int_0^a \left[ 1 - \frac{x^3}{a^3} + \frac{x^6}{a^6} \right] \, dx \]

\[ = \frac{1}{2} \left[ x - \frac{x^4}{4a^3} + \frac{x^7}{7a^6} \right]_0^a = \frac{a^2 b}{3b} \]

\[ \bar{A} = \bar{x} = \frac{3}{16} \, a^2 b \]

\[ \bar{y} = \frac{2}{5} \, a \]

\[ \bar{y} = \frac{2}{5} \, b \]

\[ \bar{A} = \bar{y} = \frac{9}{20} \, a b^2 \]

\[ \bar{y} = \frac{9}{20} \, b \]
Problem 5.1

5.1 through 5.8 Locate the centroid of the plane area shown.

\[ \overline{X} \Sigma A = \Sigma \overline{X} A \quad \overline{X}(15600) = 864 \times 10^3 \]
\[ \overline{X} = 55.4 \text{ mm} \]

\[ \overline{Y} \Sigma A = \Sigma \overline{Y} A \quad \overline{Y}(15600) = 1464 \times 10^3 \]
\[ \overline{Y} = 93.8 \text{ mm} \]

Problem 5.3

5.1 through 5.8 Locate the centroid of the plane area shown.

\[ \overline{X}(11) = 1.15 \]
\[ \overline{X} = 1.045 \text{ in.} \]

\[ \overline{Y}(11) = 3.95 \]
\[ \overline{Y} = 3.59 \text{ in.} \]
5.1 through 5.8 Locate the centroid of the plane area shown.

**Problem 5.6**

By Symmetry: \( \bar{x} = \bar{y} \)

<table>
<thead>
<tr>
<th>COMPONENT</th>
<th>( A ) mm(^2)</th>
<th>( \bar{y} ) mm</th>
<th>( \bar{z} A ) mm(^3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>SQUARE</td>
<td>( \frac{3}{4} \pi \times 75^2 = 5625 )</td>
<td>37.5</td>
<td>( 210.938 \times 10^3 )</td>
</tr>
<tr>
<td>QUARTER CIRCLE</td>
<td>( \frac{\pi}{4} \times (75)^2 - 44/128 )</td>
<td>43.169</td>
<td>( -190.716 \times 10^3 )</td>
</tr>
<tr>
<td></td>
<td>( 120.14 )</td>
<td></td>
<td>( 20.223 \times 10^3 )</td>
</tr>
</tbody>
</table>

\[ \bar{z} A = \sum \bar{z} A \]

\[ \bar{x} = \bar{y} = \bar{z} = 16.75 \text{ mm} \]

5.1 through 5.8 Locate the centroid of the plane area shown.

By Symmetry: \( \bar{y} = 0 \)

<table>
<thead>
<tr>
<th>COMPONENT</th>
<th>( A ) mm(^2)</th>
<th>( \bar{z} ) mm</th>
<th>( \bar{z} A ) mm(^3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>120-mm semicircle</td>
<td>( 4 \times 120 \times \frac{1}{2} )</td>
<td>60.93</td>
<td>-50.93</td>
</tr>
<tr>
<td>12-mm semicircle</td>
<td>( 4 \times 72 \times \frac{1}{2} )</td>
<td>-8.143</td>
<td>-30.538</td>
</tr>
<tr>
<td></td>
<td>( 104.47 \times 10^3 )</td>
<td></td>
<td>( -903.2 \times 10^3 )</td>
</tr>
</tbody>
</table>

\[ \bar{z} A = \sum \bar{z} A : \bar{z} \left( 147.7 \times 10^3 \text{ mm}^3 \right) = -903.2 \times 10^3 \text{ mm}^3 \]

\[ \bar{x} = -12.39 \text{ mm} \]

\[ \bar{z} = -62.4 \text{ mm} \]
5.51 through 5.56 Determine the reactions at the beam supports for the given loading.

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We first replace the given loading by the one shown below. Both loadings are equivalent since they are both defined by a linear relationship between load and distance and have the same values at the ends of the beam.

Each triangular loading is then replaced by a concentrated load.
Problem 5.54

5.51 through 5.56 Determine the reactions at the beam supports for the given loading.

\[ R = \frac{1}{18}(800) = 14.400 \text{ lb} \]

\[ \sum M_A = 0 \]

\[ 20B - 5(14400) = 0 \]

\[ B = 3600 \text{ lb} \]

\[ \sum F_y = 0 : A - 14400 + 3600 = 0 \]

\[ A = 10800 \text{ lb} \]

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Problem 5.55

5.51 through 5.56 Determine the reactions at the beam supports for the given loading.

\[ R_1 = 200 \text{ lb/ft}(15 \text{ ft}) \]

\[ R_1 = 3000 \text{ lb} \]

\[ R_2 = \frac{1}{2}(200 \text{ lb/ft})(6 \text{ ft}) \]

\[ R_2 = 600 \text{ lb} \]

\[ \sum M_A = 0 : -3000 \text{ lb}(1.5 \text{ ft}) - 600 \text{ lb}(9 \text{ ft} + 2 \text{ ft}) + 200 \text{ lb}(15 \text{ ft}) = 0 \]

\[ B = 740 \text{ lb} \]

\[ A = 2860 \text{ lb} \]

\[ \sum F_y = 0 : A + 740 \text{ lb} - 3000 \text{ lb} - 600 \text{ lb} = 0 \]

\[ A = 2860 \text{ lb} \]