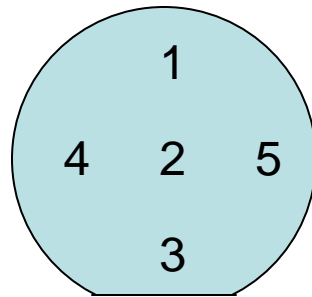


XB, R Control Chart Example

D. W. Parent

We want to learn control charts

- We have oxide thickness data generated by the Poison Diffusion Furnace Company.
- The furnace is a single wafer processor
- We have taken 5 data points per wafer



We will use the first 20 runs to set our trial control limits

Raw data

| Run | Pos 1 | Pos 2 | Pos 3 | Pos 4 | Pos 5 |
|-----|-------|-------|-------|-------|-------|
| 1 | 1008 | 954 | 1037 | 957 | 989 |
| 2 | 967 | 1009 | 1009 | 1035 | 992 |
| 3 | 1006 | 1006 | 999 | 961 | 972 |
| 4 | 1009 | 999 | 967 | 1018 | 1025 |
| 5 | 1030 | 1034 | 1000 | 992 | 974 |
| 6 | 972 | 1008 | 1002 | 1007 | 966 |
| 7 | 979 | 979 | 1031 | 1051 | 919 |
| 8 | 959 | 1011 | 1052 | 1013 | 993 |
| 9 | 898 | 1023 | 1063 | 984 | 1034 |
| 10 | 1001 | 996 | 1500 | 958 | 999 |
| 11 | 1041 | 1038 | 960 | 969 | 1013 |
| 12 | 1009 | 984 | 977 | 1047 | 1028 |
| 13 | 966 | 1006 | 987 | 1010 | 980 |
| 14 | 962 | 1036 | 1016 | 992 | 1005 |
| 15 | 1000 | 978 | 1013 | 962 | 966 |
| 16 | 1006 | 1015 | 960 | 967 | 1015 |
| 17 | 973 | 1016 | 1027 | 1000 | 1012 |
| 18 | 1000 | 999 | 1000 | 998 | 968 |
| 19 | 1026 | 1057 | 947 | 962 | 997 |
| 20 | 991 | 981 | 977 | 987 | 963 |

Calculate the average (XB) of each row.

H2 \bar{x} =AVERAGE(B2:F2)

| A | B | C | D | E | F | H |
|-----|-------|-------|-------|-------|-------|--------|
| Run | Pos 1 | Pos 2 | Pos 3 | Pos 4 | Pos 5 | XB |
| 1 | 1008 | 954 | 1037 | 957 | 989 | 989 |
| 2 | 967 | 1009 | 1009 | 1035 | 992 | 1002.4 |
| 3 | 1006 | 1006 | 999 | 961 | 972 | 988.8 |
| 4 | 1009 | 999 | 967 | 1018 | 1025 | 1003.6 |
| 5 | 1030 | 1034 | 1000 | 992 | 974 | 1006 |
| 6 | 972 | 1008 | 1002 | 1007 | 966 | 991 |
| 7 | 979 | 979 | 1031 | 1051 | 919 | 991.8 |
| 8 | 959 | 1011 | 1052 | 1013 | 993 | 1005.6 |
| 9 | 898 | 1023 | 1063 | 984 | 1034 | 1000.4 |
| 10 | 1001 | 996 | 1500 | 958 | 999 | 1090.8 |
| 11 | 1041 | 1038 | 960 | 969 | 1013 | 1004.2 |
| 12 | 1009 | 984 | 977 | 1047 | 1028 | 1009 |
| 13 | 966 | 1006 | 987 | 1010 | 980 | 989.8 |
| 14 | 962 | 1036 | 1016 | 992 | 1005 | 1002.2 |
| 15 | 1000 | 978 | 1013 | 962 | 966 | 983.8 |
| 16 | 1006 | 1015 | 960 | 967 | 1015 | 992.6 |
| 17 | 973 | 1016 | 1027 | 1000 | 1012 | 1005.6 |
| 18 | 1000 | 999 | 1000 | 998 | 968 | 993 |
| 19 | 1026 | 1057 | 947 | 962 | 997 | 997.8 |
| 20 | 991 | 981 | 977 | 987 | 963 | 979.8 |

Calculate the Range (R) of each row

| I2 | | fx =MAX(B2:F2)-MIN(B2:F2) | | | | | | |
|-----|-------|---------------------------|-------|-------|-------|--------|----|--|
| A | B | C | D | E | F | H | I | |
| Run | Pos 1 | Pos 2 | Pos 3 | Pos 4 | Pos 5 | XB | R | |
| 1 | 1008 | 954 | 1037 | 957 | 989 | 989 | 83 | |
| 2 | 967 | 1009 | 1009 | 1035 | 992 | 1002.4 | 68 | |
| 3 | 1006 | 1006 | 999 | 961 | 972 | 988.8 | 45 | |
| 4 | 1009 | 999 | 967 | 1018 | 1025 | 1003.6 | 58 | |
| 5 | 1030 | 1034 | 1000 | 992 | 974 | 1006 | 60 | |
| 6 | 972 | 1008 | 1002 | 1007 | 966 | 991 | 42 | |

Maximum value in the row minus the minimum value in the row

Calculate the Grand Mean (XBB) on the first 20 data points

| J2 fx =AVERAGE(H2:H21) | | | | | | | | |
|-----------------------------|-------|-------|-------|-------|-------|--------|-----|---------|
| A | B | C | D | E | F | H | I | J |
| Run | Pos 1 | Pos 2 | Pos 3 | Pos 4 | Pos 5 | XB | R | XBB |
| 1 | 1008 | 954 | 1037 | 957 | 989 | 989 | 83 | 1001.36 |
| 2 | 967 | 1009 | 1009 | 1035 | 992 | 1002.4 | 68 | 1001.36 |
| 3 | 1006 | 1006 | 999 | 961 | 972 | 988.8 | 45 | 1001.36 |
| 4 | 1009 | 999 | 967 | 1018 | 1025 | 1003.6 | 58 | 1001.36 |
| 5 | 1030 | 1034 | 1000 | 992 | 974 | 1006 | 60 | 1001.36 |
| 6 | 972 | 1008 | 1002 | 1007 | 966 | 991 | 42 | 1001.36 |
| 7 | 979 | 979 | 1031 | 1051 | 919 | 991.8 | 132 | 1001.36 |
| 8 | 959 | 1011 | 1052 | 1013 | 993 | 1005.6 | 93 | 1001.36 |
| 9 | 898 | 1023 | 1063 | 984 | 1034 | 1000.4 | 165 | 1001.36 |
| 10 | 1001 | 996 | 1500 | 958 | 999 | 1090.8 | 542 | 1001.36 |
| 11 | 1041 | 1038 | 960 | 969 | 1013 | 1004.2 | 81 | 1001.36 |
| 12 | 1009 | 984 | 977 | 1047 | 1028 | 1009 | 70 | 1001.36 |
| 13 | 966 | 1006 | 987 | 1010 | 980 | 989.8 | 44 | 1001.36 |
| 14 | 962 | 1036 | 1016 | 992 | 1005 | 1002.2 | 74 | 1001.36 |
| 15 | 1000 | 978 | 1013 | 962 | 966 | 983.8 | 51 | 1001.36 |
| 16 | 1006 | 1015 | 960 | 967 | 1015 | 992.6 | 55 | 1001.36 |
| 17 | 973 | 1016 | 1027 | 1000 | 1012 | 1005.6 | 54 | 1001.36 |
| 18 | 1000 | 999 | 1000 | 998 | 968 | 993 | 32 | 1001.36 |
| 19 | 1026 | 1057 | 947 | 962 | 997 | 997.8 | 110 | 1001.36 |

Cell J2 calculates the grand mean
While cell j3=j2, j4=j3.....

Calculate the mean of the Ranges (RB)

| K2 fx =AVERAGE(I2:I21) | | | | | | | | | |
|-----------------------------|-------|-------|-------|-------|-------|--------|-----|---------|-------|
| A | B | C | D | E | F | H | I | J | K |
| Run | Pos 1 | Pos 2 | Pos 3 | Pos 4 | Pos 5 | XB | R | XBB | RB |
| 1 | 1008 | 954 | 1037 | 957 | 989 | 989 | 83 | 1001.36 | 94.35 |
| 2 | 967 | 1009 | 1009 | 1035 | 992 | 1002.4 | 68 | 1001.36 | 94.35 |
| 3 | 1006 | 1006 | 999 | 961 | 972 | 988.8 | 45 | 1001.36 | 94.35 |
| 4 | 1009 | 999 | 967 | 1018 | 1025 | 1003.6 | 58 | 1001.36 | 94.35 |
| 5 | 1030 | 1034 | 1000 | 992 | 974 | 1006 | 60 | 1001.36 | 94.35 |
| 6 | 972 | 1008 | 1002 | 1007 | 966 | 991 | 42 | 1001.36 | 94.35 |
| 7 | 979 | 979 | 1031 | 1051 | 919 | 991.8 | 132 | 1001.36 | 94.35 |
| 8 | 959 | 1011 | 1052 | 1013 | 993 | 1005.6 | 93 | 1001.36 | 94.35 |
| 9 | 898 | 1023 | 1063 | 984 | 1034 | 1000.4 | 165 | 1001.36 | 94.35 |
| 10 | 1001 | 996 | 1500 | 958 | 999 | 1090.8 | 542 | 1001.36 | 94.35 |
| 11 | 1041 | 1038 | 960 | 969 | 1013 | 1004.2 | 81 | 1001.36 | 94.35 |
| 12 | 1009 | 984 | 977 | 1047 | 1028 | 1009 | 70 | 1001.36 | 94.35 |
| 13 | 966 | 1006 | 987 | 1010 | 980 | 989.8 | 44 | 1001.36 | 94.35 |
| 14 | 962 | 1036 | 1016 | 992 | 1005 | 1002.2 | 74 | 1001.36 | 94.35 |
| 15 | 1000 | 978 | 1013 | 962 | 966 | 983.8 | 51 | 1001.36 | 94.35 |
| 16 | 1006 | 1015 | 960 | 967 | 1015 | 992.6 | 55 | 1001.36 | 94.35 |
| 17 | 973 | 1016 | 1027 | 1000 | 1012 | 1005.6 | 54 | 1001.36 | 94.35 |
| 18 | 1000 | 999 | 1000 | 998 | 968 | 993 | 32 | 1001.36 | 94.35 |
| 19 | 1026 | 1057 | 947 | 962 | 997 | 997.8 | 110 | 1001.36 | 94.35 |
| 20 | 991 | 981 | 977 | 987 | 963 | 979.8 | 28 | 1001.36 | 94.35 |

Note: K3=K2.....

Calculate the Upper and Lower control limits for XB (UCLXB, LCLXB) for a sample size of 5

L2 $f_x = J2 + K2 * 0.58$

| A | B | C | D | E | F | H | I | J | K | L |
|-----|-------|-------|-------|-------|-------|--------|----|---------|-------|----------|
| Run | Pos 1 | Pos 2 | Pos 3 | Pos 4 | Pos 5 | XB | R | XBB | RB | UCLXB |
| 1 | 1008 | 954 | 1037 | 957 | 989 | 989 | 83 | 1001.36 | 94.35 | 1056.083 |
| 2 | 967 | 1009 | 1009 | 1035 | 992 | 1002.4 | 68 | 1001.36 | 94.35 | 1056.083 |
| 3 | 1006 | 1006 | 999 | 961 | 972 | 988.8 | 45 | 1001.36 | 94.35 | 1056.083 |

$$UCLXB = XBB + RB * .58$$

M2 $f_x = J2 - K2 * 0.58$

| A | B | C | D | E | F | H | I | J | K | L | M |
|-----|-------|-------|-------|-------|-------|--------|----|---------|-------|----------|---------|
| Run | Pos 1 | Pos 2 | Pos 3 | Pos 4 | Pos 5 | XB | R | XBB | RB | UCLXB | LCLXB |
| 1 | 1008 | 954 | 1037 | 957 | 989 | 989 | 83 | 1001.36 | 94.35 | 1056.083 | 946.637 |
| 2 | 967 | 1009 | 1009 | 1035 | 992 | 1002.4 | 68 | 1001.36 | 94.35 | 1056.083 | 946.637 |
| 3 | 1006 | 1006 | 999 | 961 | 972 | 988.8 | 45 | 1001.36 | 94.35 | 1056.083 | 946.637 |

$$LCLXB = XBB - RB * .58$$

Calculate the Upper Control limit for R (UCLRB) and the Lower Control limit for R (LCLRB)

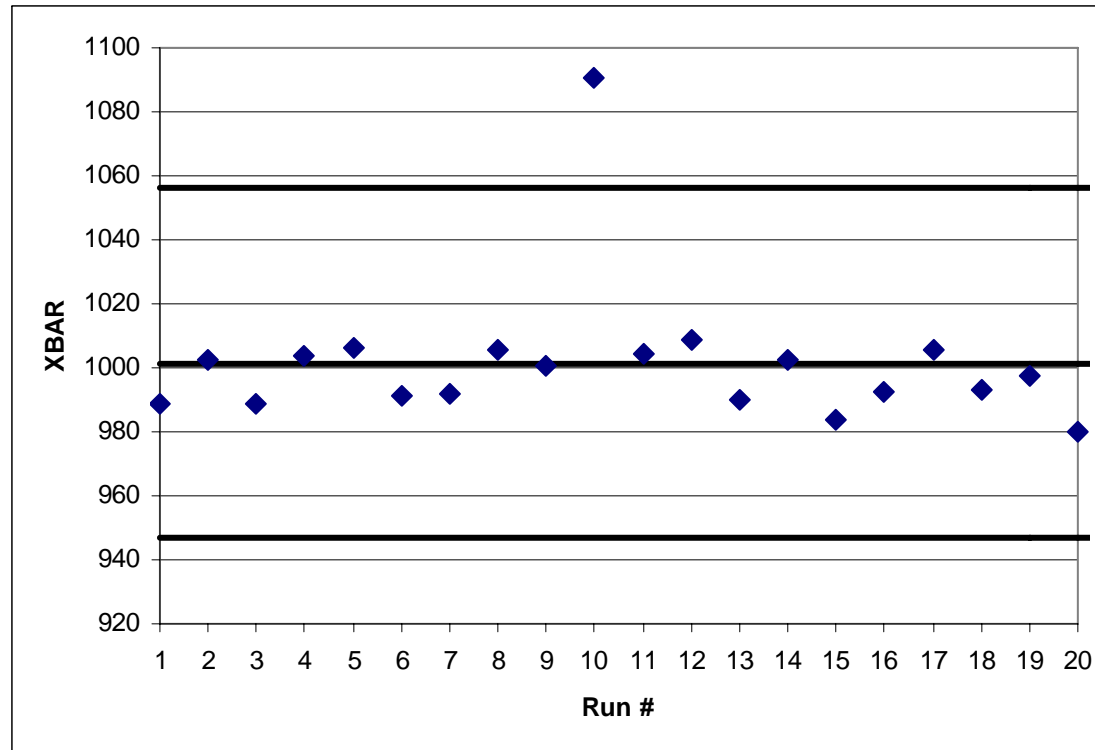
| N2 | | | | | | | | | | | | | fx =2.11*K2 |
|-----|-------|-------|-------|-------|-------|-----|----|---------|-------|----------|---------|----------|-------------|
| A | B | C | D | E | F | H | I | J | K | L | M | N | |
| Run | Pos 1 | Pos 2 | Pos 3 | Pos 4 | Pos 5 | XB | R | XBB | RB | UCLXB | LCLXB | UCLR | |
| 1 | 1008 | 954 | 1037 | 957 | 989 | 989 | 83 | 1001.36 | 94.35 | 1056.083 | 946.637 | 199.0785 | |

$UCLRB=2.11*RB$

| O2 | | | | | | | | | | | | | fx =0*K2 |
|-----|-------|-------|-------|-------|-------|--------|----|---------|-------|----------|---------|----------|----------|
| A | B | C | D | E | F | H | I | J | K | L | M | N | O |
| Run | Pos 1 | Pos 2 | Pos 3 | Pos 4 | Pos 5 | XB | R | XBB | RB | UCLXB | LCLXB | UCLR | LCLR |
| 1 | 1008 | 954 | 1037 | 957 | 989 | 989 | 83 | 1001.36 | 94.35 | 1056.083 | 946.637 | 199.0785 | 0 |
| 2 | 967 | 1009 | 1009 | 1035 | 992 | 1002.4 | 68 | 1001.36 | 94.35 | 1056.083 | 946.637 | 199.0785 | 0 |

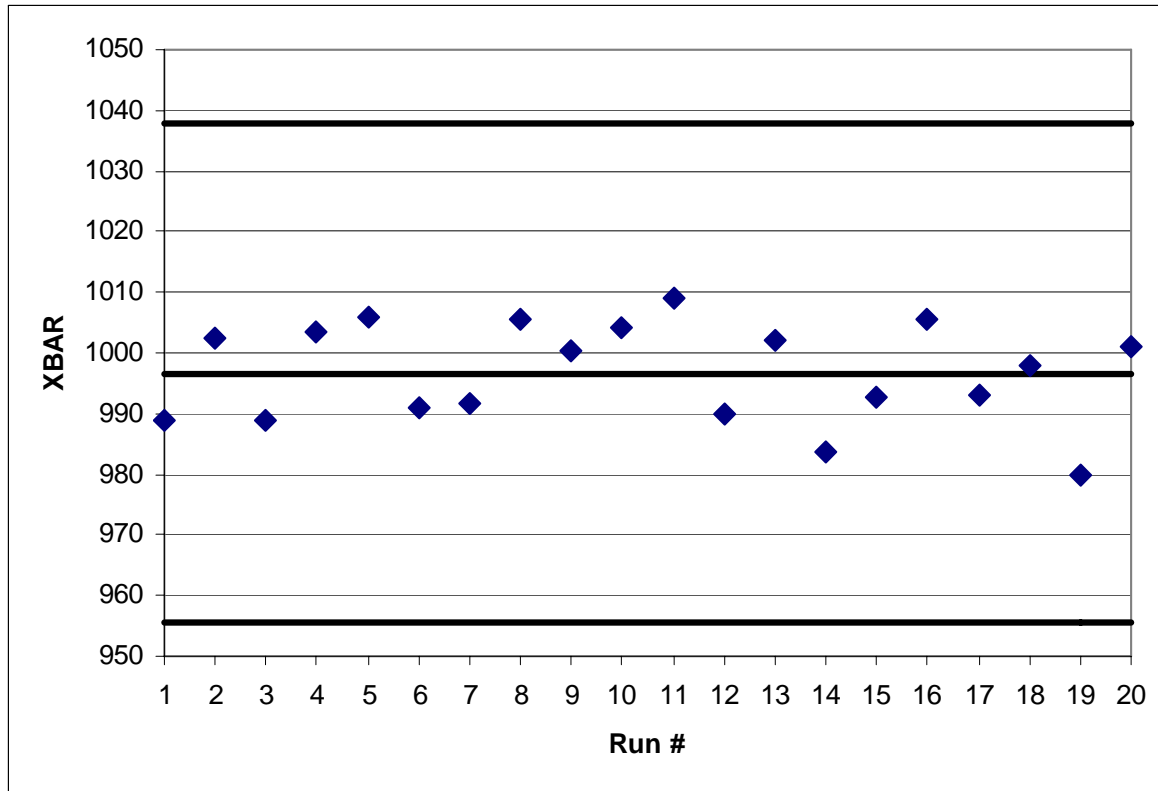
$LCLRB=0$ by definition

Plot the XB chart



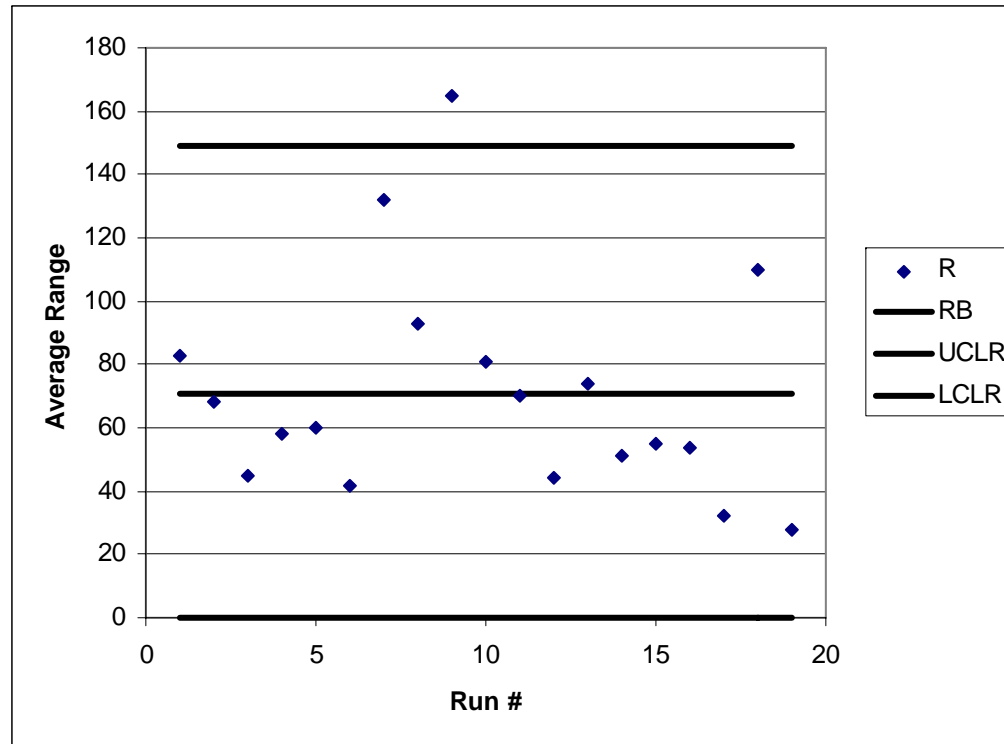
Notice we have one point outside the control limit. Maybe this was a typo when the data was entered. Remove the row that data point was from.

Re-plot the XB chart



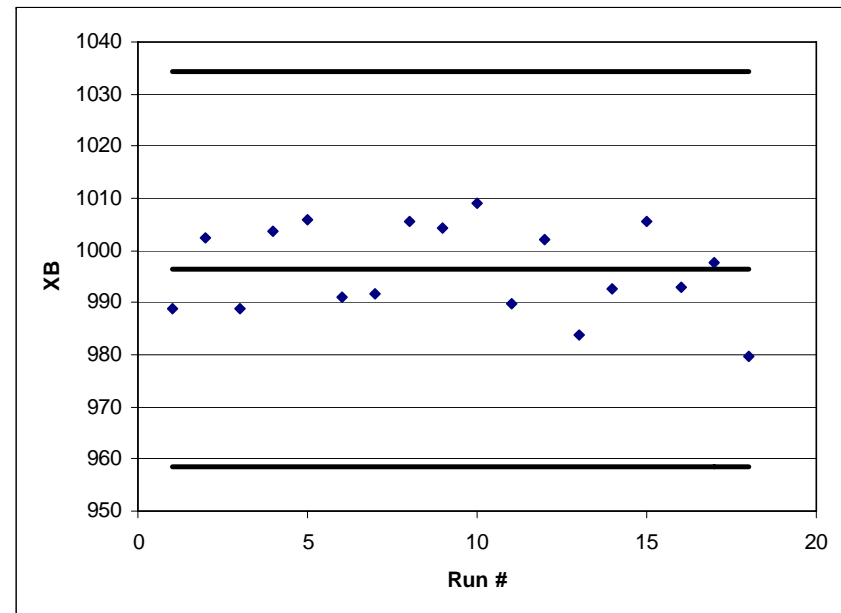
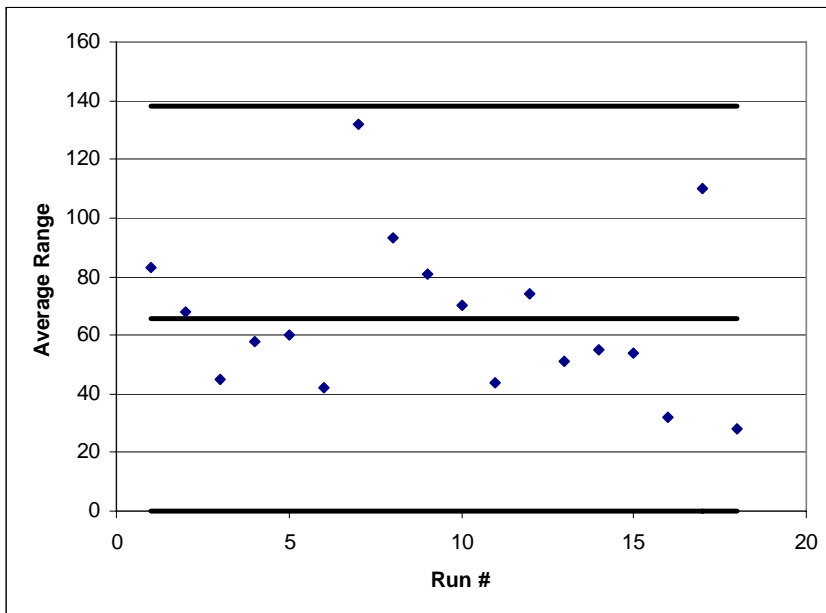
Looks good!

Plot the RB Chart



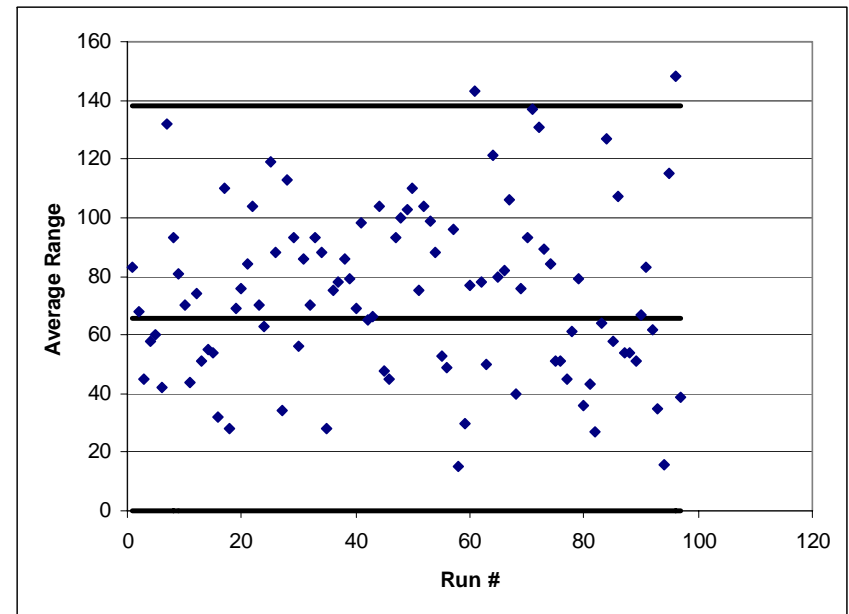
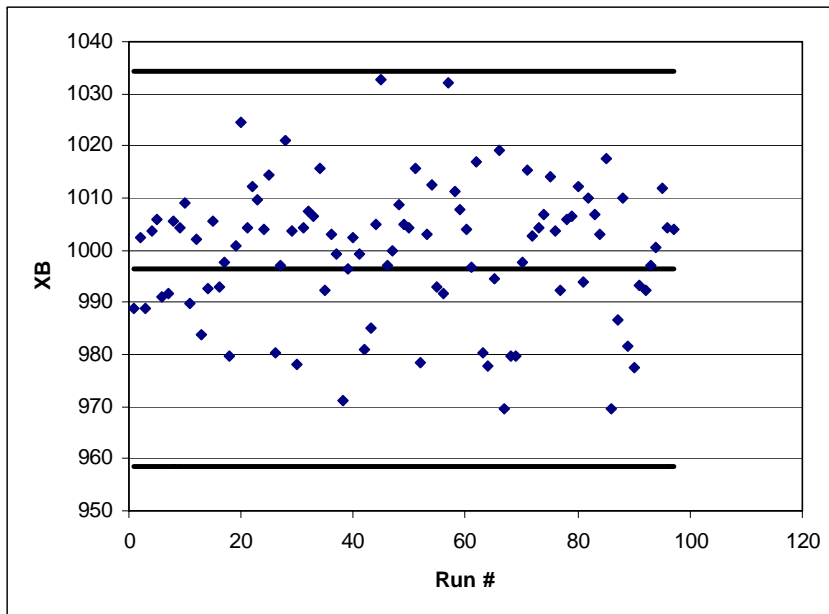
Looks like we have one bad point. Remove it and recalculate X and R charts

Trial Limits look good!



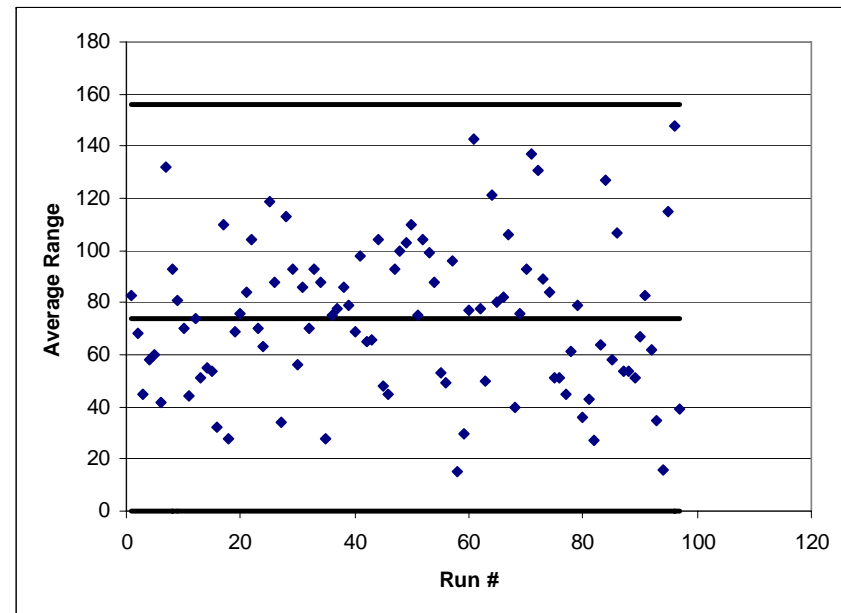
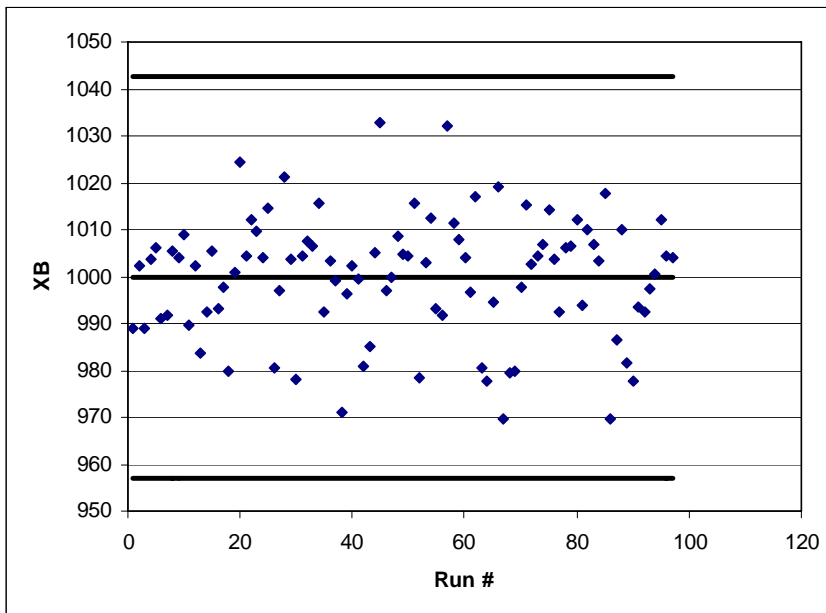
Now plot for whole data set

Final Plots?



The control limits seem to work for the XB chart, but not for the R chart. Since we did not use data from runs 21 to 100, maybe the points are not past the control limit, but the control limits are not right. Calculate control limits on the whole Set.

Final Plots?



At this time it looks like the control limits have been calculated correctly, and we Recommend that these limits be used for this process going forward.