Deriving a model to estimate WN and WP for CMOS logic gate propagation delay

Defining the linear delay model

Equations for CMOS propagation delay

The equations for propagation delay are derived in section 6.3 of Kang and Leblebici . The important assumptions are that the voltage during the input of the gate is a step response, and that the mobility is the same for the linear and saturation regions, and that there is no velocity saturation of the carriers in the channel. This model also neglects channel width narrowing. The model are restated slightly below.

$$\tau_{PHL} = \frac{L_N}{W_N \cdot K_{NP} \cdot \left(V_{DD} - V_{TN}\right)} \cdot \left[\left[2 \cdot \frac{V_{TN}}{V_{DD} - V_{TN}} + \ln\left[4 \cdot \frac{\left(V_{DD} - V_{TN}\right)}{V_{DD}} - 1 \right]\right]\right] \cdot C_{LOAD}$$

$$\tau_{PLH} = \frac{L_P}{W_P \cdot K_{PP} \cdot \left(V_{DD} + V_{TP}\right)} \cdot \left[\left[-2 \cdot \frac{V_{TP}}{V_{DD} + V_{TP}} + \ln\left[4 \cdot \frac{\left(V_{DD} + V_{TP}\right)}{V_{DD}} - 1 \right]\right]\right] \cdot C_{LOAD}$$

We can rewrite the models more simply:

$$\tau_{PHL} = \frac{L_N}{W_N} \cdot C_{LOAD} \cdot A \qquad A = \frac{1}{K_{NP} \cdot \left(V_{DD} - V_{TN}\right)} \cdot \left[\left[2 \cdot \frac{V_{TN}}{V_{DD} - V_{TN}} + \ln\left[4 \cdot \frac{\left(V_{DD} - V_{TN}\right)}{V_{DD}} - 1 \right]\right]\right]$$

$$\tau_{PHL} = \frac{L_P}{W_P} \cdot C_{LOAD} \cdot B \qquad B = \frac{1}{K_{PP} \cdot \left(V_{DD} + V_{TP}\right)} \cdot \left[\left[-2 \cdot \frac{V_{TP}}{V_{DD} + V_{TP}} + \ln\left[4 \cdot \frac{\left(V_{DD} + V_{TP}\right)}{V_{DD}} - 1 \right]\right]\right]$$

$$K_{NP} = \mu_N \cdot C_{oxide} \qquad K_{NP} = \mu_P \cdot C_{oxide}$$

Equations for the load capacitance

The equations for the load capacitance can be broken up into three parts. The output capacitance of the driver circuit, the input capacitance of the load circuit and the interconnect capacitance of the wire connecting the input to the driver.

$$C_{LOAD} = C_{OUT} + C_g + C_{int}$$

$$C_{OUT} = 2C_{gdn} + 2C_{gdp} + C_{dbn} + C_{dbp}$$

$$C_{dbn} = W_N \cdot D_{Drain} \cdot C_{JN} + 2 \cdot \left(W_N + D_{Drain}\right) \cdot C_{JSWN}$$

$$C_{dbp} = W_P \cdot D_{Drain} \cdot C_{JP} + 2 \cdot \left(W_P + D_{Drain}\right) \cdot C_{JSWP}$$

$$C_{gbp} = C_{ox} \cdot W_P \cdot L_P \quad C_{gdp} = C_{GDOP} \cdot W_P$$

Note that the WN and WP values for Cout are for the driver which in general will be different from the values of the load.

$$C_{GDON} := 1.99 \cdot 10^{-12} \frac{F}{cm}$$

$$C_{JN} := 4.23 \cdot 10^{-8} \frac{F}{cm^2}$$

$$C_{JSWN} := 3.82 \cdot 10^{-12} \frac{F}{cm}$$

$$C_{ox} := \frac{3.9 \cdot 8.85 \cdot 10^{-14} \cdot \frac{F}{cm}}{.014 \cdot 10^{-4} cm}$$

$$C_g = C_{gbn} + C_{gdn} + C_{gbp} + C_{gdp}$$

$$C_{gbn} = C_{ox} \cdot W_N \cdot L_N$$

$$C_{gbp} = C_{ox} \cdot W_P \cdot L_P$$

$$C_{gbp} = C_{ox} \cdot W_P \cdot L_P \quad C_{gdp} = C_{GDOP} \cdot W_P$$

Note that the WN and WP values for Cg are for the load which in general will be different from the values of the driver.

$$C_{ox} = 2.465 \times 10^{-7} \frac{F}{cm^2}$$

$$C_{GDOP} := 2.4 \cdot 10^{-12} \frac{F}{cm}$$

$$C_{JP} := 7.27 \cdot 10^{-8} \frac{F}{cm^2}$$

$$C_{JSWP} := 3.11 \cdot 10^{-12} \frac{F}{cm}$$

The linear delay model

Using the propagation delay equation and the equations for the load capacitance we combine them to a format of y=mx+b

$$\tau_{PHL} = \left(C_g + C_{int}\right)\cdot\frac{L_N}{W_N}\cdot A + C_{OUT}\cdot\frac{L_N}{W_N}\cdot A \quad X = \left(C_g + C_{int}\right) \qquad M = \frac{L_N}{W_N}\cdot A \quad B = C_{OUT}\cdot\frac{L_N}{W_N}\cdot A \quad Y = \tau_{PHL}$$

This model can be quite accurate if A is backfit from spice simulations to match Y=MX+B. We would like to use this modelto estimate WN and WP that will meet specification with in 20% of a spice simulation. Ideally we would like to hit timing specification for a gate with in two to three runs.

Symmetric propagation delay

In this case with want the low to high transition equation to the high to low transistion. We set the propagation delay equations equal to each other. If the lengths of the transistors are the same, and cancelling out the load capacitance we get:

$$\frac{L_P}{W_P}\cdot C_{LOAD}\cdot B = \frac{L_N}{W_N}\cdot C_{LOAD}\cdot A$$

$$W_P = W_N\cdot\frac{B}{A} \qquad R = \frac{B}{A} \qquad W_P = W_N\cdot R$$

This is only for the inverter. If we want to modify this for the worst case of a AND, NOR or AOI circuit we have to multiply LN or LP by the number of series transistors between the output node and power or ground. This assumes that there is no body effect or changes in mobility to velocity saturation.

$$\frac{N_{SP}\cdot L_P}{W_P}\cdot C_{LOAD}\cdot B = \frac{N_{SN}\cdot L_N}{W_N}\cdot C_{LOAD}\cdot A \qquad W_P = W_N\cdot\frac{B\cdot N_{SP}}{A\cdot N_{SN}} \qquad R = \frac{B\cdot N_{SP}}{A\cdot N_{SN}} \qquad W_P = W_N\cdot R$$

antisymmetric propagation delay

If we want to have antisymmetric delays we can modify the delay equations by a skew factor S

$$\frac{S \cdot N_{SP} \cdot L_P}{W_P} \cdot C_{LOAD} \cdot B = \frac{N_{SN} \cdot L_N}{W_N} \cdot C_{LOAD} \cdot A \qquad W_P = W_N \cdot \frac{S \cdot B \cdot N_{SP}}{A \cdot N_{SN}} \qquad R = \frac{S \cdot B \cdot N_{SP}}{A \cdot N_{SN}} \quad W_P = W_N \cdot R$$

We can use the relation ship of WN to WP to scale WP once we find WN

Reformulating the linear delay model in terms of WN

To make the derivation easier to read we will make some substitutions.

$$\alpha_0 = 2 \cdot D_{Dain} \cdot \left(C_{JSWN} \cdot K_{eqn} + C_{JSWP} \cdot K_{eqp}\right) + Cint + C_g$$

$$\alpha_n = K_{eqn} \cdot \left(C_{JN} \cdot D_{Drain} + 2 \cdot C_{JSWN}\right) + 2 \cdot C_{GDON}$$

$$\alpha_P = K_{eqp} \cdot \left(C_{JP} \cdot D_{Drain} + 2 \cdot C_{JSWP}\right) + 2 \cdot CGDOP$$

$$\tau_{PHL} = \frac{L_N}{W_N} \cdot \left(\alpha_0 + \alpha_n \cdot W_N + \alpha_p \cdot W_P\right) \cdot A$$

Now we can substitute WP for WNxR

$$\tau_{PHL} = \frac{L_N}{W_N} \cdot \left(\alpha_0 + \alpha_n \cdot W_N + \alpha_p \cdot R \cdot W_N\right) \cdot A$$

Solving for WN we get:

$$W_N = \frac{\alpha_0}{\frac{\tau_{PHL}}{L_N \cdot A} - \alpha_n - \alpha_p \cdot R} \qquad \text{if} \quad \alpha_n = \alpha_p \qquad W_N = \frac{\alpha_0}{\frac{\tau_{PHL}}{L_N \cdot A} - (1 + R)\alpha_n}$$

if: $C_{JSWN} \cdot K_{eqn} = C_{JSWP} \cdot K_{eqp}$

$$W_N = \frac{C_g + C_{int} + 4 \cdot D_{Drain} \cdot C_{JSWN} \cdot K_{eqn}}{\frac{\tau_{PHL}}{L_N \cdot A} - (1 + R)\alpha_n}$$

if: $K_{eqn} = 1$

$$W_N = \frac{C_g + C_{int} + 4 \cdot D_{Drain} \cdot C_{JSWN}}{\frac{\tau_{PHL}}{L_N \cdot A} - (1 + R)\left[\left(C_{JN} \cdot D_{Drain} + 2 \cdot C_{JSWN}\right) + 2 \cdot C_{GDON}\right]}$$

if: $C_{GDON} = 0$

$$W_N = \frac{C_g + C_{int} + 4 \cdot D_{Drain} \cdot C_{JSWN}}{\frac{\tau_{PHL}}{L_N \cdot A} - (1 + R)\left(C_{JN} \cdot D_{Drain} + 2 \cdot C_{JSWN}\right)}$$

Why does A and R change with propagation delay?

A is dependent on the mobility of the NMOS and the VT of the NMOS. These both change with width. It is really the width that causes a propagation delay, but in this case we are trying to find a width. B changes with width for the same reasons, but not by the same amount. Since R is the ration of B to A and since B and A change with width both A and R with change with propagation delay. What I have done is found two pairs of WN and WP that give different propagation delays and done a linear fit.One pair was the values used in the tutorial. This linear fit is not the same as the linear model. Since A and R had to be back fit,I found it easier to set the overlap capacitance equal to zero, and set the junction and side wall capacitances equal to each other.

How do I use this equation to estimate WN and WP for NAND, NOR and AOI gates?

It can be shown:

$$W_N = \frac{C_g + C_{int} + C_{JSWN}\cdot 2\cdot D_{Drain}\cdot (N + M)}{\frac{\tau_{PHL}}{N_{SN}\cdot L_N\cdot A} - (N + M\cdot R)\cdot \left(C_{JSWN}\cdot 2 + C_{JN}\cdot D_{Drain}\right)}$$

$$W_P = R\cdot W_N \qquad R = \frac{S\cdot R\cdot N_{SP}}{N_{SN}}$$

N is the number of NMOS drain capacitances in the complex gate.

M is the number of PMOS drain capacitances in the complex gate.

Example: 3 Input NAND Gate, symmetric propagation delays (.5ns) driving an external 20fF capacitance, AMI06 Technology

$S := 1 \quad C_g := 20\cdot 10^{-15}F \quad C_{int} := 0F \quad \tau_{PHL} := .5\cdot 10^{-9}s \quad D_{Drain} := 1.5\cdot 10^{-4}cm$

$N_{SP} := 1 \quad N_{SN} := 3 \quad N := 5 \quad M := 3 \quad L_N := .6\cdot 10^{-4}cm$

$m1 := -5\cdot 10^{12}\cdot \frac{\Omega}{s} \quad m2 := -250\cdot 10^{6}Hz \quad b_1 := 12.8\cdot 10^{3}\Omega \quad b_2 := 1.83$

$A := m1\cdot \tau_{PHL} + b_1 \quad R := m2\cdot \tau_{PHL} + b_2 \quad R := \frac{S\cdot R\cdot N_{SP}}{N_{SN}} \quad A = 1.03\times 10^{4}\Omega \quad R = 0.568$

$$W_N := \frac{C_g + C_{int} + C_{JSWN}\cdot 2\cdot D_{Drain}\cdot (N+M)}{\frac{\tau_{PHL}}{N_{SN}\cdot L_N\cdot A} - (N + M\cdot R)\cdot \left(C_{JSWN}\cdot 2 + C_{JN}\cdot D_{Drain}\right)} \qquad W_N = 1.658\times 10^{-4}cm$$

$W_P := R\cdot W_N \quad W_P = 9.423\times 10^{-5}cm$

WP is non physical, so we set it to 1.5um and find WN

$W_P := 1.5\cdot 10^{-4}cm \quad W_N := \frac{W_P}{R} \quad W_N = 2.639\times 10^{-4}cm$

This will produce symmetric delays but faster than we wanted. We could just set WP tot he minimum and use the WN value we calculated.

$\tau_{PLH} := \tau_{PHL}$

$$\tau_{PHL\_Measured} := (347 - 50)\cdot 10^{-12}s \qquad Error := \frac{\tau_{PHL\_Measured} - \tau_{PHL}}{\tau_{PHL}}\cdot 100 \qquad Error = -40.6$$

$$\tau_{PLH\_Measured} := (2654 - 2150)\cdot 10^{-12}s \qquad Error := \frac{\tau_{PLH\_Measured} - \tau_{PLH}}{\tau_{PLH}}\cdot 100 \qquad Error = 0.8$$

$$W_N := \frac{\tau_{PHL\_Measured}}{\tau_{PHL}}\cdot W_N \qquad W_N = 1.568\times 10^{-4}cm$$

$$W_P := \frac{\tau_{PLH\_Measured}}{\tau_{PLH}}\cdot W_P \qquad W_P = 1.512\times 10^{-4}cm$$

$$\tau_{PHL\_Measured} := (563 - 50)\cdot 10^{-12}s \qquad Error := \frac{\tau_{PHL\_Measured} - \tau_{PHL}}{\tau_{PHL}}\cdot 100 \qquad Error = 2.6$$

$$\tau_{PLH\_Measured} := (2548 - 2150)\cdot 10^{-12}s \qquad Error := \frac{\tau_{PLH\_Measured} - \tau_{PLH}}{\tau_{PLH}}\cdot 100 \qquad Error = -20.4$$

$$W_N := \frac{\tau_{PHL\_Measured}}{\tau_{PHL}} \cdot W_N \qquad W_N = 1.609 \times 10^{-4} \, cm$$

$$W_P := \frac{\tau_{PLH\_Measured}}{\tau_{PLH}} \cdot W_P \qquad W_P = 1.204 \times 10^{-4} \, cm$$

Notice that WP is non-physical. We can never get a symmetric propagation delay of .5ns under these conditions.
In fact in the ami06 technology WN would round to 1.5um and so would WP!

Example: 3 Input NAND Gate, symmetric propagation delays (.5ns) driving an external 40fF capacitance, AMI06 Technology

$S := 1 \quad C_g := 40 \cdot 10^{-15} F \quad C_{int} := 0F \quad \tau_{PHL} := .4 \cdot 10^{-9} s \quad D_{Drain} := 1.5 \cdot 10^{-4} cm$

$N_{SP} := 1 \quad N_{SN} := 3 \quad N := 5 \quad M := 3 \quad L_N := .6 \cdot 10^{-4} cm$

$m1 := -5 \cdot 10^{12} \cdot \frac{\Omega}{s} \quad m2 := -250 \cdot 10^{6} Hz \quad b_1 := 12.8 \cdot 10^{3} \Omega \quad b_2 := 1.83$

$A := m1 \cdot \tau_{PHL} + b_1 \quad R := m2 \cdot \tau_{PHL} + b_2 \quad R := \frac{S \cdot R \cdot N_{SP}}{N_{SN}} \quad A = 1.08 \times 10^{4} \Omega \quad R = 0.577$

$$W_N := \frac{C_g + C_{int} + C_{JSWN} \cdot 2 \cdot D_{Drain} \cdot (N + M)}{\frac{\tau_{PHL}}{N_{SN} \cdot L_N \cdot A} - (N + M \cdot R) \cdot \left(C_{JSWN} \cdot 2 + C_{JN} \cdot D_{Drain}\right)} \qquad W_N = 4.404 \times 10^{-4} cm$$

$W_P := R \cdot W_N \quad W_P = 2.54 \times 10^{-4} cm$

$\tau_{PLH} := \tau_{PHL}$

$$\tau_{PHL\_Measured} := (335 - 50)\cdot 10^{-12}s \qquad Error := \frac{\tau_{PHL\_Measured} - \tau_{PHL}}{\tau_{PHL}}\cdot 100 \qquad Error = -28.75$$

$$\tau_{PLH\_Measured} := (2614 - 2150)\cdot 10^{-12}s \qquad Error := \frac{\tau_{PLH\_Measured} - \tau_{PLH}}{\tau_{PLH}}\cdot 100 \qquad Error = 16$$

$$W_N := \frac{\tau_{PHL\_Measured}}{\tau_{PHL}}\cdot W_N \qquad W_N = 3.138\times 10^{-4}cm$$

$$W_P := \frac{\tau_{PLH\_Measured}}{\tau_{PLH}}\cdot W_P \qquad W_P = 2.946\times 10^{-4}cm$$

$$\tau_{PHL\_Measured} := (438 - 50)\cdot 10^{-12}s \qquad Error := \frac{\tau_{PHL\_Measured} - \tau_{PHL}}{\tau_{PHL}}\cdot 100 \qquad Error = -3$$

$$\tau_{PLH\_Measured} := (2551 - 2150)\cdot 10^{-12}s \qquad Error := \frac{\tau_{PLH\_Measured} - \tau_{PLH}}{\tau_{PLH}}\cdot 100 \qquad Error = 0.25$$

We hit the spec in two spice runs! WN actually rounds to 3.15u, and WP rounds to 3u. Notice the true ratio has changed quite a bit from the one we calculated. This is due to the body effect and less velocity saturation in the nmos stack which leads to an increased drive ability, We could recalculate R and A for a 3 input NAND and use them when ever we need to design a NAND.

$$R := \frac{W_P}{W_N}$$

$$R = 0.939$$

$S := 1 \quad C_g := 40\cdot 10^{-15}F \quad C_{int} := 0F \quad \tau_{PHL} := .4\cdot 10^{-9}s \quad D_{Drain} := 1.5\cdot 10^{-4}cm$

$N_{SP} := 1 \quad N_{SN} := 3 \quad N := 5 \quad M := 3 \quad L_N := .6\cdot 10^{-4}cm$

$A := 8373\Omega \quad R = 0.939$ Vary A until you get WN=3.15um notice that WP is 2.957u. This would be fine because the amio6 technology file would round it to 3um.

$$W_N := \frac{C_g + C_{int} + C_{JSWN}\cdot 2\cdot D_{Drain}\cdot (N+M)}{\frac{\tau_{PHL}}{N_{SN}\cdot L_N\cdot A} - (N + M\cdot R)\cdot \left(C_{JSWN}\cdot 2 + C_{JN}\cdot D_{Drain}\right)}$$

$W_N = 3.15\times 10^{-4}cm$

$W_P := R\cdot W_N \quad W_P = 2.957\times 10^{-4}cm$

Lets try one more with our new values of A and R.

$S := 1 \quad C_g := 20 \cdot 10^{-15} F \quad C_{int} := 0F \quad \tau_{PHL} := .25 \cdot 10^{-9} s \quad D_{Drain} := 1.5 \cdot 10^{-4} cm$

$N_{SP} := 1 \quad N_{SN} := 3 \quad N := 5 \quad M := 3 \quad L_N := .6 \cdot 10^{-4} cm$

$A := 8373\Omega \quad R = 0.939$

$$W_N := \frac{C_g + C_{int} + C_{JSWN} \cdot 2 \cdot D_{Drain} \cdot (N + M)}{\frac{\tau_{PHL}}{N_{SN} \cdot L_N \cdot A} - (N + M \cdot R) \cdot \left(C_{JSWN} \cdot 2 + C_{JN} \cdot D_{Drain}\right)}$$

$W_N = 5.157 \times 10^{-4} cm$

$W_P := R \cdot W_N \quad W_P = 4.841 \times 10^{-4} cm$

$\tau_{PLH} := \tau_{PHL}$

$$\tau_{PHL\_Measured} := (283 - 50) \cdot 10^{-12} s \qquad Error := \frac{\tau_{PHL\_Measured} - \tau_{PHL}}{\tau_{PHL}} \cdot 100 \qquad Error = -6.8$$

$$\tau_{PLH\_Measured} := (2366 - 2150) \cdot 10^{-12} s \qquad Error := \frac{\tau_{PLH\_Measured} - \tau_{PLH}}{\tau_{PLH}} \cdot 100 \qquad Error = -13.6$$

$$W_N := \frac{\tau_{PHL\_Measured}}{\tau_{PHL}} \cdot W_N \qquad W_N = 4.806 \times 10^{-4} cm$$

$$W_P := \frac{\tau_{PLH\_Measured}}{\tau_{PLH}} \cdot W_P \qquad W_P = 4.183 \times 10^{-4} cm$$

$$\tau_{PHL\_Measured} := (284 - 50) \cdot 10^{-12} s \qquad Error := \frac{\tau_{PHL\_Measured} - \tau_{PHL}}{\tau_{PHL}} \cdot 100 \qquad Error = -6.4$$

$$\tau_{PLH\_Measured} := (2387 - 2150) \cdot 10^{-12} s \qquad Error := \frac{\tau_{PLH\_Measured} - \tau_{PLH}}{\tau_{PLH}} \cdot 100 \qquad Error = -5.2$$

You still need two spice simulations. Well if it takes two runs anyway why bother finding a new value for A and R? If we used the old values for A and R we would have gotten a negative number for a .25ns propagation delay! The model would have told us that it would be impossible to go this fast.

What about the TSMC25 process?

$A := 6150\Omega$ $\quad C_{JSWN} := 4.44\cdot 10^{-12}\frac{F}{cm}$ $\quad C_{JN} := 1.92\cdot 10^{-7}\frac{F}{cm^2}$ $\quad C_{GDO} := 6.27\cdot 10^{-12}\frac{F}{cm}$

$L_N := .24\cdot 10^{-4}cm$ $\quad K_{EQ} := \frac{-2\sqrt{.99}}{-2.5}\cdot\left(\sqrt{.99+2.5}-\sqrt{.99}\right)$ $\quad K_{EQ} = 0.695$ $\quad D_D := .6\cdot 10^{-4}cm$

$C_g := 10\cdot 10^{-15}F$ $\quad N := 5$ $\quad N_{SN} := 3$

$\tau_{PHL} := .2\cdot 10^{-9}s$ $\quad M := 3$ $\quad N_{SP} := 1$ $\quad S := 1$

$R := 2.322\cdot\frac{N_{SP}\cdot S}{N_{SN}}$ $\quad R = 0.774$

$$W_N := \frac{C_g + C_{JSWN}\cdot 2\cdot D_D\cdot K_{EQ}\cdot(N+M)}{\frac{\tau_{PHL}}{N_{SN}\cdot L_N\cdot A} - (N + M\cdot R)\cdot\left(C_{JSWN}\cdot 2K_{EQ} + C_{JN}\cdot D_D K_{EQ} + 2\cdot C_{GDO}\right)}$$

$W_N = 5.063\times 10^{-5}cm$ Note: Any W values below 360nm are non-physical solutions.

$W_P := R\cdot W_N$ $\quad W_P = 3.919\times 10^{-5}cm$

$$\tau_{PHL\_Measured} := (189 - 15)\cdot 10^{-12} s \qquad Error := \frac{\tau_{PHL\_Measured} - \tau_{PHL}}{\tau_{PHL}}\cdot 100 \qquad Error = -13$$

$$\tau_{PLH\_Measured} := (2305 - 2025)\cdot 10^{-12} s \qquad Error := \frac{\tau_{PLH\_Measured} - \tau_{PLH}}{\tau_{PLH}}\cdot 100 \qquad Error = 12$$

$$W_N := \frac{\tau_{PHL\_Measured}}{\tau_{PHL}}\cdot W_N \qquad W_N = 4.405\times 10^{-5} cm$$

420

$$W_P := \frac{\tau_{PLH\_Measured}}{\tau_{PLH}}\cdot W_P \qquad W_P = 4.389\times 10^{-5} cm$$

$$\tau_{PHL\_Measured} := (208 - 15)\cdot 10^{-12} s \qquad Error := \frac{\tau_{PHL\_Measured} - \tau_{PHL}}{\tau_{PHL}}\cdot 100 \qquad Error = -3.5$$

$$\tau_{PLH\_Measured} := (2273 - 2025)\cdot 10^{-12} s \qquad Error := \frac{\tau_{PLH\_Measured} - \tau_{PLH}}{\tau_{PLH}}\cdot 100 \qquad Error = -0.8$$

In this case WN and WP both round to 420nm!

Example: Domino Gates.

We have 2 stages of domino logic. For the inverter:

$A := 6150\Omega$ $\quad C_{JSWN} := 4.44\cdot10^{-12}\frac{F}{cm}$ $\quad C_{JN} := 1.92\cdot10^{-7}\frac{F}{cm^2}$ $\quad C_{GDO} := 6.27\cdot10^{-12}\frac{F}{cm}$

$L_N := .24\cdot10^{-4}cm$ $\quad K_{EQ} := \frac{-2\sqrt{.99}}{-2.5}\cdot\left(\sqrt{.99+2.5}-\sqrt{.99}\right)$ $\quad K_{EQ} = 0.695$ $\quad D_D := .6\cdot10^{-4}cm$

$C_g := 10\cdot10^{-15}F$ $\quad N := 1$ $\quad N_{SN} := 1$

$\tau_{PHL} := .1\cdot10^{-9}s$ $\quad M := 1$ $\quad N_{SP} := 1$ $\quad S := 2$ $\quad R := 2.322\cdot\frac{N_{SP}\cdot S}{N_{SN}}$ $\quad R = 4.644$

$$W_N := \frac{C_g + C_{JSWN}\cdot2\cdot D_D\cdot K_{EQ}\cdot(N+M)}{\frac{\tau_{PHL}}{N_{SN}\cdot L_N\cdot A} - (N + M\cdot R)\cdot\left(C_{JSWN}\cdot2K_{EQ} + C_{JN}\cdot D_D K_{EQ} + 2\cdot C_{GDO}\right)}$$

$W_N = 2.039\times10^{-5}cm$ Note: Any W values below 360nm are non-physical solutions.

$W_P := R\cdot W_N$ $\quad W_P = 1.672\times10^{-4}cm$ $\quad W_N := 3.6\cdot10^{-5}cm$ $\quad W_P := R\cdot W_N$ $\quad W_P = 1.672\times10^{-4}cm$

$$\tau_{PLH} := \tau_{PHL} \cdot \frac{1}{2}$$

$$\tau_{PHL\_Measured} := (116 - 15) \cdot 10^{-12} s \qquad Error := \frac{\tau_{PHL\_Measured} - \tau_{PHL}}{\tau_{PHL}} \cdot 100 \qquad Error = 1$$

$$\tau_{PLH\_Measured} := (2086 - 2025) \cdot 10^{-12} s \qquad Error := \frac{\tau_{PLH\_Measured} - \tau_{PLH}}{\tau_{PLH}} \cdot 100 \qquad Error = 22$$

$$\tau_{PLH\_Measured} = 6.1 \times 10^{-11} s$$

$$W_N := \frac{\tau_{PHL\_Measured}}{\tau_{PHL}} \cdot W_N \qquad W_N = 3.636 \times 10^{-5} cm$$

$$W_P := \frac{\tau_{PLH\_Measured}}{\tau_{PLH}} \cdot W_P \qquad W_P = 2.04 \times 10^{-4} cm$$

$$\tau_{PHL\_Measured} := (124 - 15) \cdot 10^{-12} s \qquad Error := \frac{\tau_{PHL\_Measured} - \tau_{PHL}}{\tau_{PHL}} \cdot 100 \qquad Error = 9$$

$$\tau_{PLH\_Measured} := (2078 - 2025) \cdot 10^{-12} s \qquad Error := \frac{\tau_{PLH\_Measured} - \tau_{PLH}}{\tau_{PLH}} \cdot 100 \qquad Error = 6$$

$$W_N := \frac{\tau_{PHL\_Measured}}{\tau_{PHL}} \cdot W_N \qquad W_N = 3.963 \times 10^{-5}\,cm$$

$$W_P := \frac{\tau_{PLH\_Measured}}{\tau_{PLH}} \cdot W_P \qquad W_P = 2.162 \times 10^{-4}\,cm$$

WN rounds to 420n and WP rounds to 2.16u

$$\tau_{PHL\_Measured} := (118 - 15)\cdot 10^{-12}s \qquad Error := \frac{\tau_{PHL\_Measured} - \tau_{PHL}}{\tau_{PHL}} \cdot 100 \qquad Error = 3$$

$$\tau_{PLH\_Measured} := (2077 - 2025)\cdot 10^{-12}s \qquad Error := \frac{\tau_{PLH\_Measured} - \tau_{PLH}}{\tau_{PLH}} \cdot 100 \qquad Error = 4$$

$A := 6150\Omega$ $\quad C_{JSWN} := 4.44\cdot10^{-12}\frac{F}{cm}$ $\quad C_{JN} := 1.92\cdot10^{-7}\frac{F}{cm^2}$ $\quad C_{GDO} := 6.27\cdot10^{-12}\frac{F}{cm}$

$L_N := .24\cdot10^{-4}cm$ $\quad K_{EQ} := \frac{-2\sqrt{.99}}{-2.5}\cdot\left(\sqrt{.99+2.5}-\sqrt{.99}\right)$ $\quad K_{EQ} = 0.695$ $\quad D_D := .6\cdot10^{-4}cm$

$C_g := 9.38\cdot10^{-15}F$ $\quad N := 3$ $\quad N_{SN} := 2$

$\tau_{PHL} := .1\cdot10^{-9}s$ $\quad M := 1$ $\quad N_{SP} := 1$ $\quad S := \frac{1}{2}$ $\quad R := 2.322\cdot\frac{N_{SP}\cdot S}{N_{SN}}$ $\quad R = 0.58$

$$W_N := \frac{C_g + C_{JSWN}\cdot 2\cdot D_D\cdot K_{EQ}\cdot(N+M)}{\frac{\tau_{PHL}}{N_{SN}\cdot L_N\cdot A} - (N + M\cdot R)\cdot\left(C_{JSWN}\cdot 2K_{EQ} + C_{JN}\cdot D_D K_{EQ} + 2\cdot C_{GDO}\right)}$$

$W_N = 4.468\times10^{-5}cm$ $\quad$ Note: Any W values below 360nm are non-physical solutions.

$W_P := R\cdot W_N$ $\quad W_P = 2.594\times10^{-5}cm$ $\quad W_P := 3.6\cdot10^{-5}cm$ $\quad W_N := \frac{W_P}{R}$ $\quad W_N = 7.697\times10^{-5}cm$

$$\tau_{PLH} := \tau_{PHL} \cdot 2$$

$$\tau_{PHL\_Measured} := (102 - 15) \cdot 10^{-12} s \qquad Error := \frac{\tau_{PHL\_Measured} - \tau_{PHL}}{\tau_{PHL}} \cdot 100 \qquad Error = -13$$

$$\tau_{PLH\_Measured} := (2311 - 2025) \cdot 10^{-12} s \qquad Error := \frac{\tau_{PLH\_Measured} - \tau_{PLH}}{\tau_{PLH}} \cdot 100 \qquad Error = 43$$

$$W_N := \frac{\tau_{PHL\_Measured}}{\tau_{PHL}} \cdot W_N \qquad W_N = 6.696 \times 10^{-5} cm$$

$$W_P := \frac{\tau_{PLH\_Measured}}{\tau_{PLH}} \cdot W_P \qquad W_P = 3.709 \times 10^{-5} cm$$

$$\tau_{PHL\_Measured} := (108 - 15) \cdot 10^{-12} s \qquad Error := \frac{\tau_{PHL\_Measured} - \tau_{PHL}}{\tau_{PHL}} \cdot 100 \qquad Error = -7$$

$$\tau_{PLH\_Measured} := (2290 - 2025) \cdot 10^{-12} s \qquad Error := \frac{\tau_{PLH\_Measured} - \tau_{PLH}}{\tau_{PLH}} \cdot 100 \qquad Error = 32.5$$

$$W_N := \frac{\tau_{PHL\_Measured}}{\tau_{PHL}} \cdot W_N \qquad W_N = 6.228 \times 10^{-5} cm$$

$$W_P := \frac{\tau_{PLH\_Measured}}{\tau_{PLH}} \cdot W_P \qquad W_P = 4.914 \times 10^{-5} cm$$

$$\tau_{PHL\_Measured} := (114 - 15)\cdot 10^{-12}s \qquad Error := \frac{\tau_{PHL\_Measured} - \tau_{PHL}}{\tau_{PHL}}\cdot 100 \qquad Error = -1$$

$$\tau_{PLH\_Measured} := (2233 - 2025)\cdot 10^{-12}s \qquad Error := \frac{\tau_{PLH\_Measured} - \tau_{PLH}}{\tau_{PLH}}\cdot 100 \qquad Error = 4$$

WN rounds to 600nm and WP rounds to 480nm

$A := 6150\Omega$ $C_{JSWN} := 4.44\cdot 10^{-12}\frac{F}{cm}$ $C_{JN} := 1.92\cdot 10^{-7}\frac{F}{cm^2}$ $C_{GDO} := 6.27\cdot 10^{-12}\frac{F}{cm}$

$L_N := .24\cdot 10^{-4}cm$ $K_{EQ} := \frac{-2\sqrt{.99}}{-2.5}\cdot\left(\sqrt{.99+2.5}-\sqrt{.99}\right)$ $K_{EQ} = 0.695$ $D_D := .6\cdot 10^{-4}cm$

$C_g := 1.5\cdot 10^{-15}F$ $N := 1$ $N_{SN} := 1$

$\tau_{PHL} := .1\cdot 10^{-9}s$ $M := 1$ $N_{SP} := 1$ $S := 2$ $R := 2.322\cdot\frac{N_{SP}\cdot S}{N_{SN}}$ $R = 4.644$

$$W_N := \frac{C_g + C_{JSWN}\cdot 2\cdot D_D\cdot K_{EQ}\cdot(N+M)}{\frac{\tau_{PHL}}{N_{SN}\cdot L_N\cdot A} - (N + M\cdot R)\cdot\left(C_{JSWN}\cdot 2K_{EQ} + C_{JN}\cdot D_D K_{EQ} + 2\cdot C_{GDO}\right)}$$

$W_N = 4.254\times 10^{-6}cm$ Note: Any W values below 360nm are non-physical solutions.

$W_P := R\cdot W_N$ $W_P = 1.976\times 10^{-5}cm$ $W_P := 3.6\cdot 10^{-5}cm$ $W_N := \frac{W_N}{R}$ $W_N = 9.16\times 10^{-7}cm$

Note: This is still non-physical set WN=WP=360n

$W_N := .360\cdot 10^{-4}cm$ $W_P := W_N$

$$\tau_{PLH} := \tau_{PHL} \cdot \frac{1}{2}$$

$$\tau_{PHL\_Measured} := (43 - 15) \cdot 10^{-12} s \qquad Error := \frac{\tau_{PHL\_Measured} - \tau_{PHL}}{\tau_{PHL}} \cdot 100 \qquad Error = -72$$

$$\tau_{PLH\_Measured} := (2089 - 2025) \cdot 10^{-12} s \qquad Error := \frac{\tau_{PLH\_Measured} - \tau_{PLH}}{\tau_{PLH}} \cdot 100 \qquad Error = 28$$

$$W_N := \frac{\tau_{PHL\_Measured}}{\tau_{PHL}} \cdot W_N \qquad W_N = 1.008 \times 10^{-5} cm \qquad W_N := .360 \cdot 10^{-4} cm$$

$$W_P := \frac{\tau_{PLH\_Measured}}{\tau_{PLH}} \cdot W_P \qquad W_P = 4.608 \times 10^{-5} cm$$

$\tau_{PHL\_Measured} := (45 - 15) \cdot 10^{-12} s$ $\quad Error := \frac{\tau_{PHL\_Measured} - \tau_{PHL}}{\tau_{PHL}} \cdot 100$ $\quad Error = -70$

$\tau_{PLH\_Measured} := (2080 - 2025) \cdot 10^{-12} s$ $\quad Error := \frac{\tau_{PLH\_Measured} - \tau_{PLH}}{\tau_{PLH}} \cdot 100$ $\quad Error = 10$

$W_N := \frac{\tau_{PHL\_Measured}}{\tau_{PHL}} \cdot W_N$ $\quad W_N = 1.08 \times 10^{-5} cm$ $\quad W_N := .360 \cdot 10^{-4} cm$

$W_P := \frac{\tau_{PLH\_Measured}}{\tau_{PLH}} \cdot W_P$ $\quad W_P = 5.069 \times 10^{-5} cm$

$\tau_{PHL\_Measured} := (46 - 15) \cdot 10^{-12} s$ $\quad Error := \frac{\tau_{PHL\_Measured} - \tau_{PHL}}{\tau_{PHL}} \cdot 100$ $\quad Error = -69$

$\tau_{PLH\_Measured} := (2077 - 2025) \cdot 10^{-12} s$ $\quad Error := \frac{\tau_{PLH\_Measured} - \tau_{PLH}}{\tau_{PLH}} \cdot 100$ $\quad Error = 4$

I just picked WP=600n
WN=360n

$\tau_{PHL\_Measured} := (48 - 15) \cdot 10^{-12} s$ $\quad Error := \frac{\tau_{PHL\_Measured} - \tau_{PHL}}{\tau_{PHL}} \cdot 100$ $\quad Error = -67$

$\tau_{PLH\_Measured} := (2074 - 2025) \cdot 10^{-12} s$ $\quad Error := \frac{\tau_{PLH\_Measured} - \tau_{PLH}}{\tau_{PLH}} \cdot 100$ $\quad Error = -2$

$A := 6150\Omega$ $C_{JSWN} := 4.44\cdot 10^{-12}\frac{F}{cm}$ $C_{JN} := 1.92\cdot 10^{-7}\frac{F}{cm^2}$ $C_{GDO} := 6.27\cdot 10^{-12}\frac{F}{cm}$

$L_N := .24\cdot 10^{-4}cm$ $K_{EQ} := \frac{-2\sqrt{.99}}{-2.5}\cdot\left(\sqrt{.99+2.5}-\sqrt{.99}\right)$ $K_{EQ} = 0.695$ $D_D := .6\cdot 10^{-4}cm$

$C_g := 3.5\cdot 10^{-15}F$ $N := 5$ $N_{SN} := 3$

$\tau_{PHL} := .1\cdot 10^{-9}s$ $M := 1$ $N_{SP} := 1$ $S := \frac{1}{2}$ $R := 2.322\cdot\frac{N_{SP}\cdot S}{N_{SN}}$ $R = 0.387$

$$W_N := \frac{C_g + C_{JSWN}\cdot 2\cdot D_D\cdot K_{EQ}\cdot(N+M)}{\frac{\tau_{PHL}}{N_{SN}\cdot L_N\cdot A} - (N + M\cdot R)\cdot\left(C_{JSWN}\cdot 2K_{EQ} + C_{JN}\cdot D_D K_{EQ} + 2\cdot C_{GDO}\right)}$$

$W_N = 6.986\times 10^{-5}cm$ Note: Any W values below 360nm are non-physical solutions.

$W_P := R\cdot W_N$ $W_P = 2.704\times 10^{-5}cm$

$\tau_{PLH} := 2\cdot\tau_{PHL}$ $W_P := .36\cdot 10^{-4}cm$ $W_N := \frac{W_P}{R}$ $W_N = 9.302\times 10^{-5}cm$

$$\tau_{PHL\_Measured} := (101 - 15)\cdot 10^{-12} s \qquad Error := \frac{\tau_{PHL\_Measured} - \tau_{PHL}}{\tau_{PHL}}\cdot 100 \quad Error = -14$$

$$\tau_{PLH\_Measured} := (2304 - 2025)\cdot 10^{-12} s \quad Error := \frac{\tau_{PLH\_Measured} - \tau_{PLH}}{\tau_{PLH}}\cdot 100 \quad Error = 39.5$$

$$W_N := \frac{\tau_{PHL\_Measured}}{\tau_{PHL}}\cdot W_N \qquad W_N = 8\times 10^{-5} cm$$

$$W_P := \frac{\tau_{PLH\_Measured}}{\tau_{PLH}}\cdot W_P \qquad W_P = 5.022\times 10^{-5} cm$$

$$\tau_{PHL\_Measured} := (108 - 15)\cdot 10^{-12} s \qquad Error := \frac{\tau_{PHL\_Measured} - \tau_{PHL}}{\tau_{PHL}}\cdot 100 \quad Error = -7$$

$$\tau_{PLH\_Measured} := (2218 - 2025)\cdot 10^{-12} s \quad Error := \frac{\tau_{PLH\_Measured} - \tau_{PLH}}{\tau_{PLH}}\cdot 100 \quad Error = -3.5$$

WN rounded to 780n and WP to 480n

$\tau_{PLH} := .4 \cdot 10^{-9} s$ $\quad \tau_{PHL} := .2 \cdot 10^{-9} s$

$\tau_{PLH\_Measured} := (336 - 15) \cdot 10^{-12} s$ $\quad Error := \frac{\tau_{PHL\_Measured} - \tau_{PHL}}{\tau_{PHL}} \cdot 100$ $\quad Error = -53.5$

$\tau_{PLH\_Measured} = 3.21 \times 10^{-10} s$

$\tau_{PHL\_Measured} := (2373 - 2025) \cdot 10^{-12} s$ $\quad Error := \frac{\tau_{PLH\_Measured} - \tau_{PLH}}{\tau_{PLH}} \cdot 100$ $\quad Error = -19.75$

$\tau_{PHL\_Measured} = 3.48 \times 10^{-10} s$

$A := 6150\Omega$ $\quad C_{JSWN} := 4.44\cdot 10^{-12}\frac{F}{cm}$ $\quad C_{JN} := 1.92\cdot 10^{-7}\frac{F}{cm^2}$ $\quad C_{GDO} := 6.27\cdot 10^{-12}\frac{F}{cm}$

$L_N := .24\cdot 10^{-4}cm$ $\quad K_{EQ} := \frac{-2\sqrt{.99}}{-2.5}\cdot\left(\sqrt{.99+2.5}-\sqrt{.99}\right)$ $\quad K_{EQ} = 0.695$ $\quad D_D := .6\cdot 10^{-4}cm$

$C_g := 10\cdot 10^{-15}F$ $\quad N := 1$ $\quad N_{SN} := 1$

$\tau_{PHL} := .05\cdot 10^{-9}s$ $\quad M := 1$ $\quad N_{SP} := 1$ $\quad S := 1$ $\quad R := 2.322\cdot\frac{N_{SP}\cdot S}{N_{SN}}$ $\quad R = 2.322$

$$W_N := \frac{C_g + C_{JSWN}\cdot 2\cdot D_D\cdot K_{EQ}\cdot(N+M)}{\frac{\tau_{PHL}}{N_{SN}\cdot L_N\cdot A} - (N + M\cdot R)\cdot\left(C_{JSWN}\cdot 2K_{EQ} + C_{JN}\cdot D_D K_{EQ} + 2\cdot C_{GDO}\right)}$$

$W_N = 4.296\times 10^{-5}cm$ $\quad$ Note: Any W values below 360nm are non-physical solutions.

$W_P := R\cdot W_N$ $\quad W_P = 9.976\times 10^{-5}cm$

$W_N := .36\cdot 10^{-4}cm$ $\quad W_P := .36\cdot 10^{-4}cm$

$A := 6150\Omega$ $\quad C_{JSWN} := 4.44 \cdot 10^{-12} \frac{F}{cm}$ $\quad C_{JN} := 1.92 \cdot 10^{-7} \frac{F}{cm^2}$ $\quad C_{GDO} := 6.27 \cdot 10^{-12} \frac{F}{cm}$

$L_N := .24 \cdot 10^{-4} cm$ $\quad K_{EQ} := \frac{-2\sqrt{.99}}{-2.5} \cdot \left(\sqrt{.99 + 2.5} - \sqrt{.99}\right)$ $\quad K_{EQ} = 0.695$ $\quad D_D := .6 \cdot 10^{-4} cm$

$C_g := 5.29 \cdot 10^{-15} F$ $\quad N := 8$ $\quad N_{SN} := 3$

$\tau_{PHL} := .15 \cdot 10^{-9} s$ $\quad M := 1$ $\quad N_{SP} := 1$ $\quad S := 1$ $\quad R := 2.322 \cdot \frac{N_{SP} \cdot S}{N_{SN}}$ $\quad R = 0.774$

$$W_N := \frac{C_g + C_{JSWN} \cdot 2 \cdot D_D \cdot K_{EQ} \cdot (N + M)}{\frac{\tau_{PHL}}{N_{SN} \cdot L_N \cdot A} - (N + M \cdot R) \cdot \left(C_{JSWN} \cdot 2K_{EQ} + C_{JN} \cdot D_D K_{EQ} + 2 \cdot C_{GDO}\right)}$$

$W_N = 8.265 \times 10^{-5} cm$ Note: Any W values below 360nm are non-physical solutions.

$W_P := R \cdot W_N$ $\quad W_P = 6.397 \times 10^{-5} cm$

$W_N := .36 \cdot 10^{-4} cm$ $\quad W_P := W_N$

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