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Question 1 (20PTS):

Design a MOS capacitor to have a VT of .25 Volts, when the substrate is Silicon, doped p-Type 10^{17}cm^{-3}, and the oxide layer has a dielectric constant of $3.9\varepsilon_o$. Assume that the gate is made out of N+ poly-silicon, room temperature, and the Qi is
$1\text{x}10^{11}\text{q/cm}^2$.

$$T := 300 \quad k := 1.38\cdot10^{-23} \quad q := 1.6\cdot10^{-19}$$

$$U_T := \frac{k\cdot T}{q} \qquad V_T := .25$$

$$\chi := 4.05 \quad \phi_n := 0 \quad E_g := 1.12 \quad n_i := 1.5\cdot10^{10}$$

$$\phi_m := \chi + \phi_n \qquad N_A := 10^{17}$$

$$\phi_p := U_T\cdot\ln\left(\frac{n_i}{N_A}\right) \qquad \phi_p = -0.407$$

$$\varepsilon_{SiO2} := 3.9\cdot8.85\cdot10^{-14}$$

$$\varepsilon_{Si} := 11.8\cdot8.85\cdot10^{-14}$$

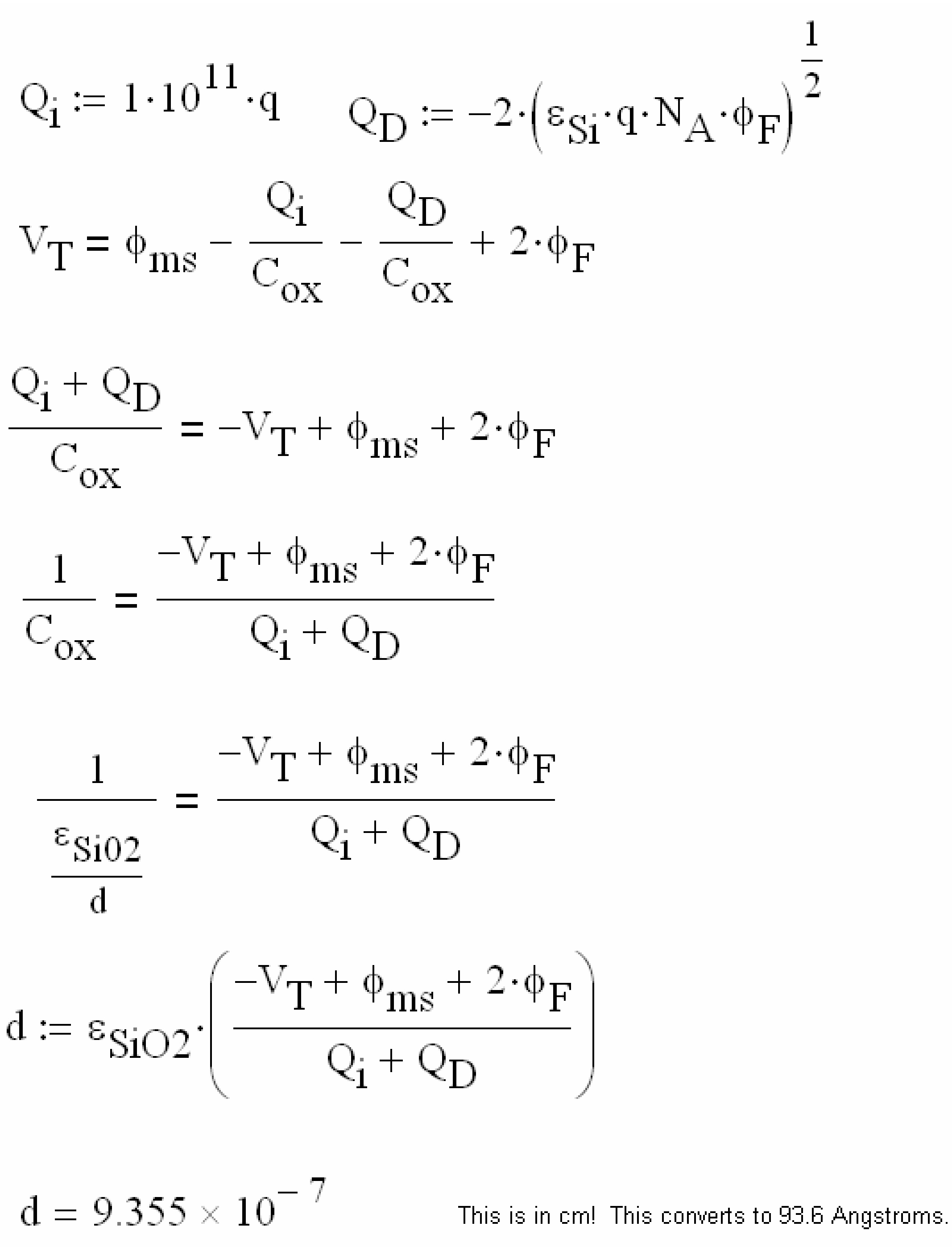
$$\phi_{ms} := \phi_m - \left(\chi + \frac{E_g}{2} - \phi_p\right)$$

$$\phi_{ms} := \chi - \chi - \frac{E_g}{2} + \phi_p$$

$$\phi_{ms} := \frac{-E_g}{2} + \phi_p \qquad \phi_{ms} = -0.967$$

$$\phi_F := U_T\cdot\ln\left(\frac{N_A}{n_i}\right) \qquad \phi_F = 0.407$$

NAME:



$$Q_i := 1\cdot10^{11}\cdot q \qquad Q_D := -2\cdot\left(\varepsilon_{Si}\cdot q\cdot N_A\cdot\phi_F\right)^{\frac{1}{2}}$$

$$V_T = \phi_{ms} - \frac{Q_i}{C_{ox}} - \frac{Q_D}{C_{ox}} + 2\cdot\phi_F$$

$$\frac{Q_i + Q_D}{C_{ox}} = -V_T + \phi_{ms} + 2\cdot\phi_F$$

$$\frac{1}{C_{ox}} = \frac{-V_T + \phi_{ms} + 2\cdot\phi_F}{Q_i + Q_D}$$

$$\frac{1}{\frac{\varepsilon_{Si02}}{d}} = \frac{-V_T + \phi_{ms} + 2\cdot\phi_F}{Q_i + Q_D}$$

$$d := \varepsilon_{SiO2}\cdot\left(\frac{-V_T + \phi_{ms} + 2\cdot\phi_F}{Q_i + Q_D}\right)$$

$$d = 9.355 \times 10^{-7}$$

This is in cm! This converts to 93.6 Angstroms.

NAME:

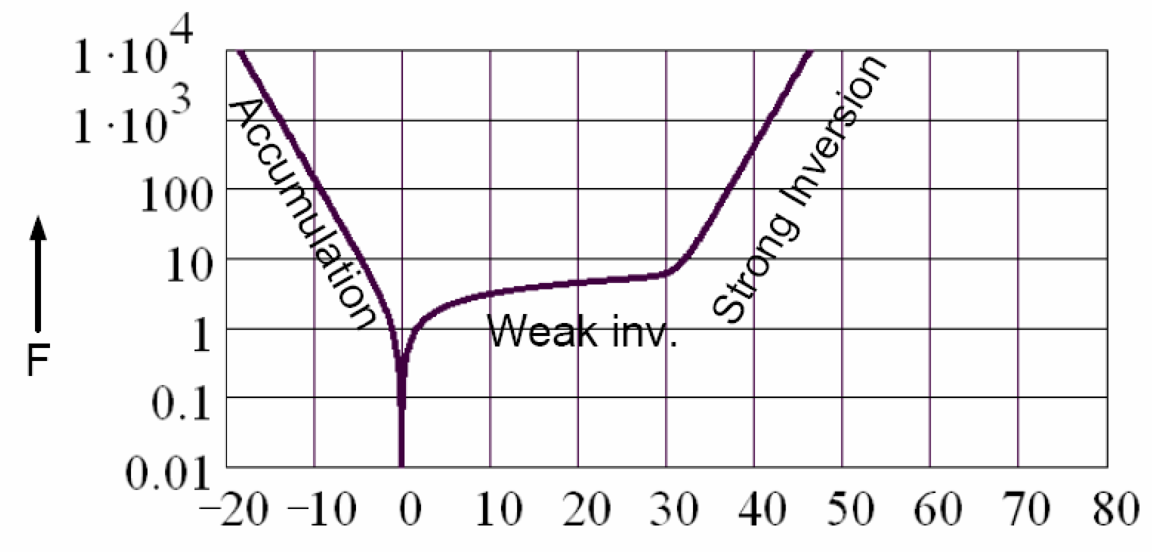
Question 2(20pts):

Please explain why we use MOS capacitors in accumulation mode for on chip capacitors.

Before process engineers added a thin oxide layer between two metals, the layer with the most per unit area capacitance was all ways the gate oxide layer. Given that circuit designers want to minimize area, and that signals vary in amplitude over time, the MIS capacitor is used to minimize the area.

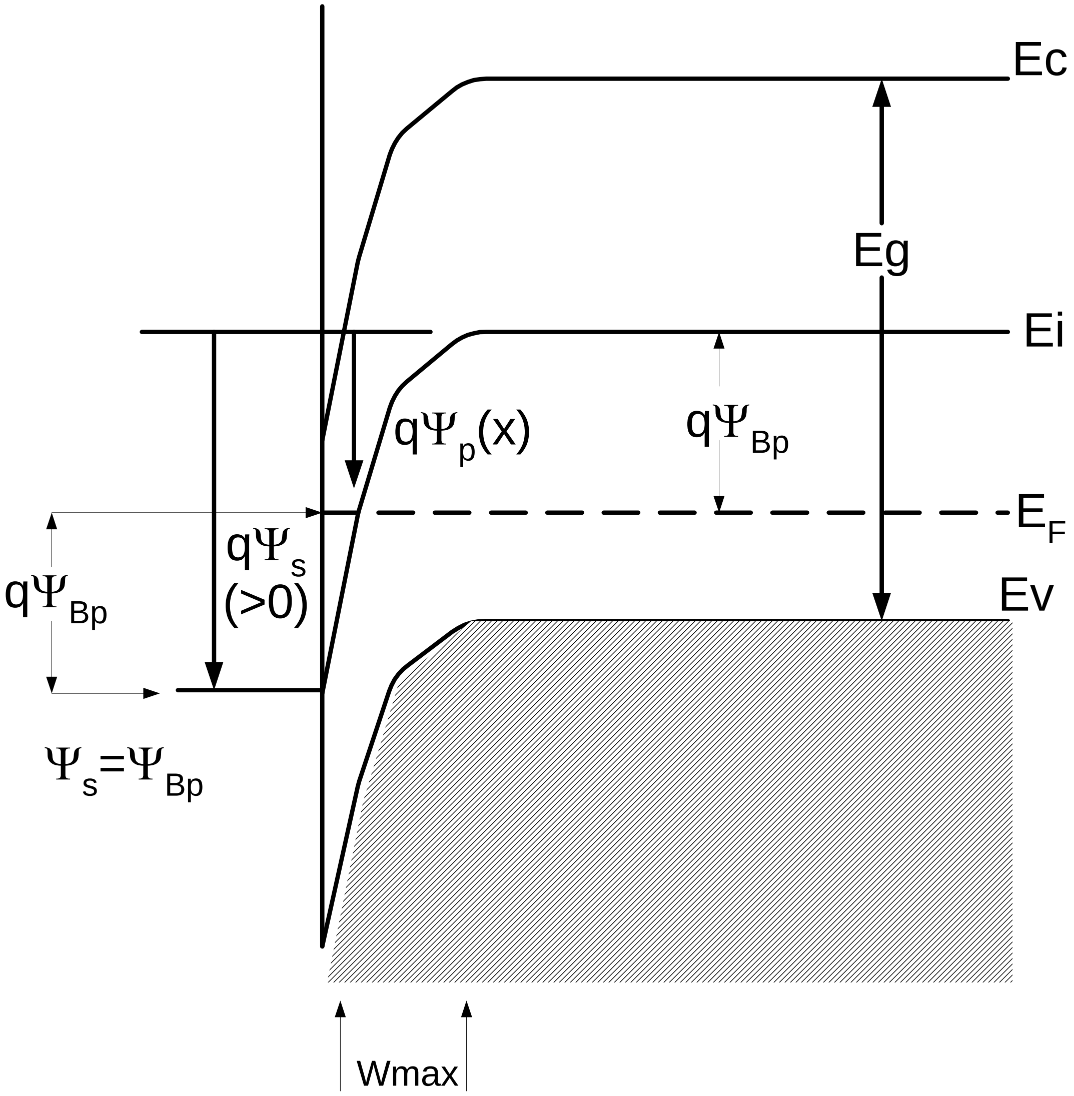
Why accumulation? As we know the MIS capacitor has two capacitors in series, the gate oxide capacitance and the depletion capacitance. When the charge due to the change accumulated in the substrate is large, so is the capacitance, and the depletion capacitance drops out of the equation. This leaves the gate oxide capacitance which is relatively independent of the voltage applied.

If we look at the function F, and realize that the capacitance in the substrate is the derivative of the function F, We see in accumulation the derivative is large, thus the capacitance is large.

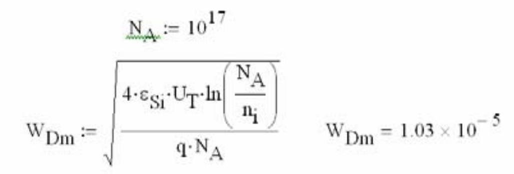


Question 3 (20pts):

Please draw and Energy band diagram of an MIS capacitor, with NA=1x10^{17}cm^{-3} (Si) at threshold. Label everything correctly, and calculate band parameters.



Wmax=



$N_A := 10^{17}$

$$W_{Dm} := \sqrt{\frac{4 \cdot \varepsilon_{Si} \cdot U_T \cdot \ln\left(\frac{N_A}{n_i}\right)}{q \cdot N_A}} \qquad W_{Dm} = 1.03 \times 10^{-5}$$

ΨB=.407eV
EG=1.12Ev

Question 4(25pts):

What is the temperature dependence of the threshold voltage of an MIS capacitor be given that the substrate is Silicon, doped p-Type 10^{17}cm^{-3}, and the oxide layer has a dielectric constant of $3.9\varepsilon_o$. Assume that the gate is made out of N+ poly-silicon, and the Qi is $5\text{x}10^{10}$q/cm^2.

First let's rearrange the metal semiconductor work function:

$$\phi_{ms} = \frac{-E_g}{2} + \phi_p$$

$$\phi_{ms} = \frac{-E_g}{2} - \phi_F$$

$$\phi_F = U_T \cdot \ln\left(\frac{N_A}{n_i}\right)$$

Then let's expand this into the VT equation:

$$V_T = \frac{-E_g}{2} - \phi_F - \frac{Q_i}{C_{ox}} - \frac{-2\cdot\left(\varepsilon_{Si}\cdot q\cdot N_A\cdot \phi_F\right)^{\frac{1}{2}}}{C_{ox}} + 2\cdot\phi_F$$

$$V_T = \frac{-E_g}{2} - \frac{Q_i}{C_{ox}} - \frac{-2\cdot\left(\varepsilon_{Si}\cdot q\cdot N_A\cdot \phi_F\right)^{\frac{1}{2}}}{C_{ox}} + 1\cdot\phi_F$$

Use the chain Rule:

Equation 1:

$$\frac{d}{dT}V_T = \frac{d}{dT}\phi_F\cdot\left(1 + \frac{1}{C_{ox}}\cdot\sqrt{\frac{\varepsilon_{Si}\cdot q\cdot N_A}{\phi_F}}\right)$$

Now we need to find:

$$\frac{d}{dT}\phi_F$$

$$\phi_F = \frac{k\cdot T}{q}\cdot\ln\left(\frac{N_A}{3.16\cdot 10^{16}\cdot T^{1.5}\cdot e^{\frac{-E_g\cdot q}{2k\cdot T}}}\right)$$

$$\phi_F = \frac{k\cdot T}{q}\cdot\left(\ln\left(\frac{N_A}{3.16\cdot 10^{16}}\right) - 1.5\cdot\ln(T) - \ln\left(e^{\frac{-E_g\cdot q}{2k\cdot T}}\right)\right)$$

$$\phi_F = \frac{k\cdot T}{q}\cdot\ln\left(\frac{N_A}{3.16\cdot 10^{16}}\right) - 1.5\cdot\frac{k\cdot T}{q}\cdot\ln(T) - \frac{k\cdot T}{q}\cdot\ln\left(e^{\frac{-E_g\cdot q}{2k\cdot T}}\right)$$

$$\frac{d}{dT}\phi_F = \frac{k}{q}\cdot\ln\left(\frac{N_A}{3.16\cdot 10^{16}}\right) - 1.5\cdot\frac{k}{T}\cdot\ln(T) - 1.5\cdot\frac{k\cdot T}{q}\cdot\frac{1}{T} - \frac{k}{q}\cdot\ln\left(e^{\frac{-E_g\cdot q}{2k\cdot T}}\right) - \frac{k\cdot T}{q}\cdot\frac{-E_g\cdot q}{2k\cdot T^2}$$

$$\frac{d}{dT}\phi_F = \frac{k}{q}\cdot\ln\left(\frac{N_A}{3.16\cdot 10^{16}}\right) - 1.5\cdot\frac{k}{T}\cdot\ln(T) - 1.5\cdot\frac{k\cdot T}{q}\cdot\frac{1}{T} - \frac{k}{q}\cdot\ln\left(e^{\frac{-E_g\cdot q}{2k\cdot T}}\right) + \frac{-E_g}{2\cdot T}$$

$$\frac{d}{dT}\phi_F = \frac{k}{q}\cdot\ln\left(\frac{N_A}{3.16\cdot 10^{16}\cdot T^{1.5}\cdot e^{\frac{-E_g\cdot q}{k\cdot T}}}\right) - 1.5\cdot\frac{k}{q} + \frac{E_g}{2\cdot T}$$

$$\frac{d}{dT}\phi_F = \frac{1}{T}\left(\frac{k\cdot T}{q}\cdot\ln\left(\frac{N_A}{3.16\cdot 10^{16}\cdot T^{1.5}\cdot e^{\frac{-E_g\cdot q}{k\cdot T}}}\right) - 1.5\cdot\frac{k\cdot T}{q} + \frac{E_g}{2}\right)$$

$$\frac{d}{dT}\phi_F = \frac{1}{T}\left(\phi_F - 1.5\cdot\frac{k\cdot T}{q} + \frac{E_g}{2}\right)$$

Combine this last result with equation 1:

$$\frac{d}{dT}V_T = \frac{1}{T}\cdot\left(U_T\cdot\ln\left(\frac{N_A}{3.16\cdot10^{16}\cdot T^{1.5}\cdot e^{\frac{-E_g}{2U_T}}}\right)-\frac{E_g}{2}-1.5\cdot U_T\right)\cdot\left(1+\frac{1}{C_{ox}}\cdot\sqrt{\frac{q\cdot\varepsilon_{Si}\cdot N_A}{U_T\cdot\ln\left(\frac{N_A}{3.16\cdot10^{16}\cdot T^{1.5}\cdot e^{\frac{-E_g}{2\cdot U_T}}}\right)}}\right)$$

MathCad can calculate a numerical derivative. As can be seen below the analytical VT (T) is the same:

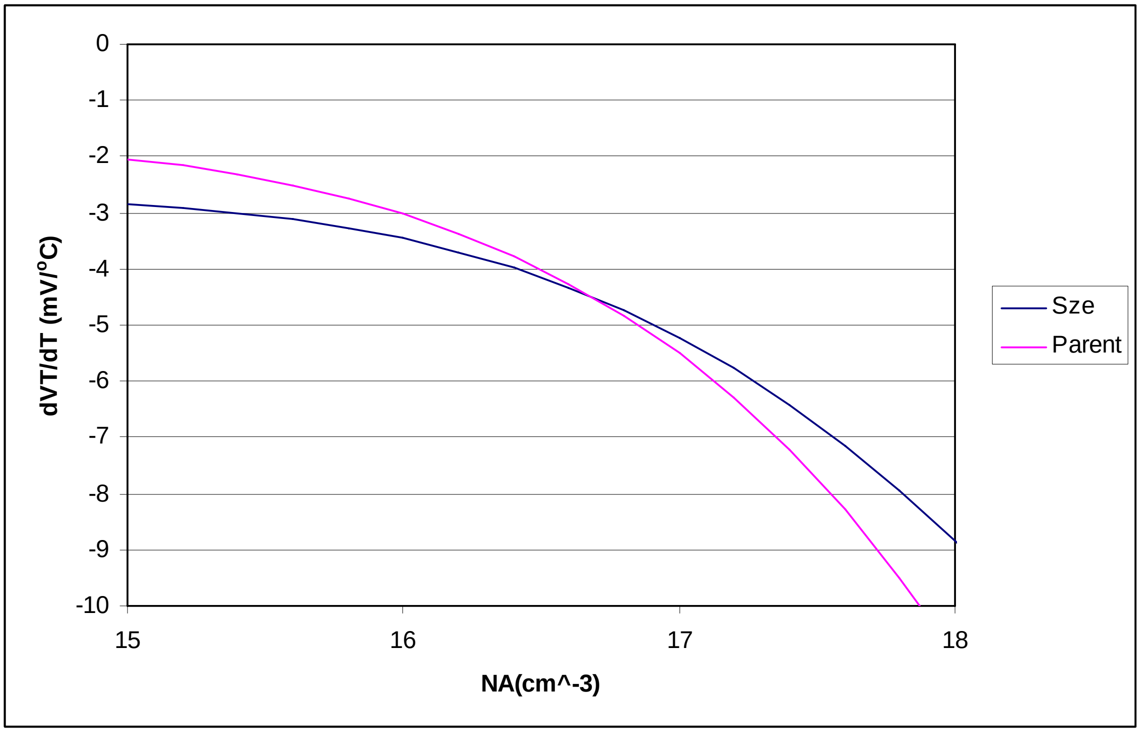
$$\frac{d}{dT}\left[\frac{-E_g}{2}-\frac{Q_i}{C_{ox}}-\frac{-2\cdot\left[\varepsilon_{Si}\cdot q\cdot N_A\cdot\left(k\cdot\frac{T}{q}\cdot\ln\left(\frac{N_A}{3.16\cdot10^{16}\cdot T^{1.5}\cdot e^{\frac{-E_g\cdot q}{2\cdot k\cdot T}}}\right)\right)\right]^{\frac{1}{2}}}{C_{ox}}+1\cdot\left(k\cdot\frac{T}{q}\cdot\ln\left(\frac{N_A}{3.16\cdot10^{16}\cdot T^{1.5}\cdot e^{\frac{-E_g\cdot q}{2\cdot k\cdot T}}}\right)\right)\right]=-1.401\times10^{-3}$$

$$\frac{1}{T}\cdot\left(k\cdot\frac{T}{q}\cdot\ln\left(\frac{N_A}{3.16\cdot10^{16}\cdot T^{1.5}\cdot e^{\frac{-E_g\cdot q}{2\cdot k\cdot T}}}\right)-\frac{E_g}{2}-1.5\cdot U_T\right)\cdot\left(1+\frac{1}{C_{ox}}\cdot\sqrt{\frac{q\cdot\varepsilon_{Si}\cdot N_A}{k\cdot\frac{T}{q}\cdot\ln\left(\frac{N_A}{3.16\cdot10^{16}\cdot T^{1.5}\cdot e^{\frac{-E_g\cdot q}{2\cdot k\cdot T}}}\right)}}\right)=-1.401\times10^{-3}$$

Note: This is on page 319 of “Big Sze” with the following simplification:

$$\frac{d}{dT}V_T = \frac{1}{T}\cdot\left(U_T\cdot\ln\left(\frac{N_A}{3.16\cdot10^{16}\cdot T^{1.5}\cdot e^{\frac{-E_g}{2U_T}}}\right)-\frac{E_g}{2}\right)\cdot\left(2+\frac{1}{C_{ox}}\cdot\sqrt{\frac{q\cdot\varepsilon_{Si}\cdot N_A}{U_T\cdot\ln\left(\frac{N_A}{3.16\cdot10^{16}\cdot T^{1.5}\cdot e^{\frac{-E_g}{2\cdot U_T}}}\right)}}\right)$$

If we plot the two equations we get:

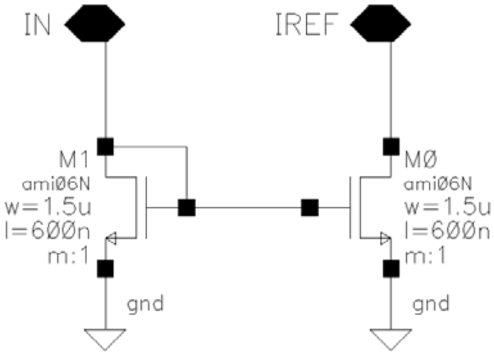


Given the at most it is 1mV per degree C of error the simplification seems fine.

NAME:

Question 5(15pts):

You are a process engineer and you are choosing the substrate doping level (NA). The source and drain doping levels are already selected to be Nd=10^{19}cm^{-3}. In order to insure that IREF matches IN as closely as possible (assume the VT mismatch is zero), should you choose as high a level of substrate doping as possible or as low a substrate value as possible? Justify your answer.



Channel width modulation causes ID to be dependant on VD is the saturation mode. The smaller the depletion width, then smaller the depletion width changes with reverse bias voltage, and thus the less ID increases. If ID=μnCoxW/2L x (VGS-VT2)^2(1+VDSλ), this is seen as a decrease in l. Given that M1 is at one fixed VD and that M0's VD can vary, a smaller λ, means that the ID varies less, and thus mirrors M1 more closely. The substrate I would choose would be as high as I could make it (which reduces the depletion width), realistically 10^{17}cm^{-3}.