

#1 Find the small signal RF model of a MOSFET.

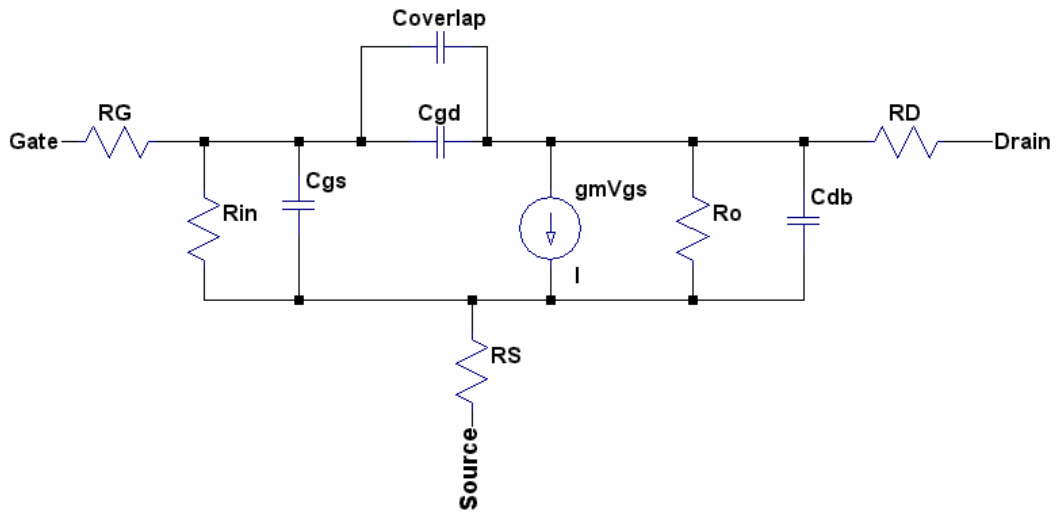


Figure 1: Small Signal RF model for a MOSFET.

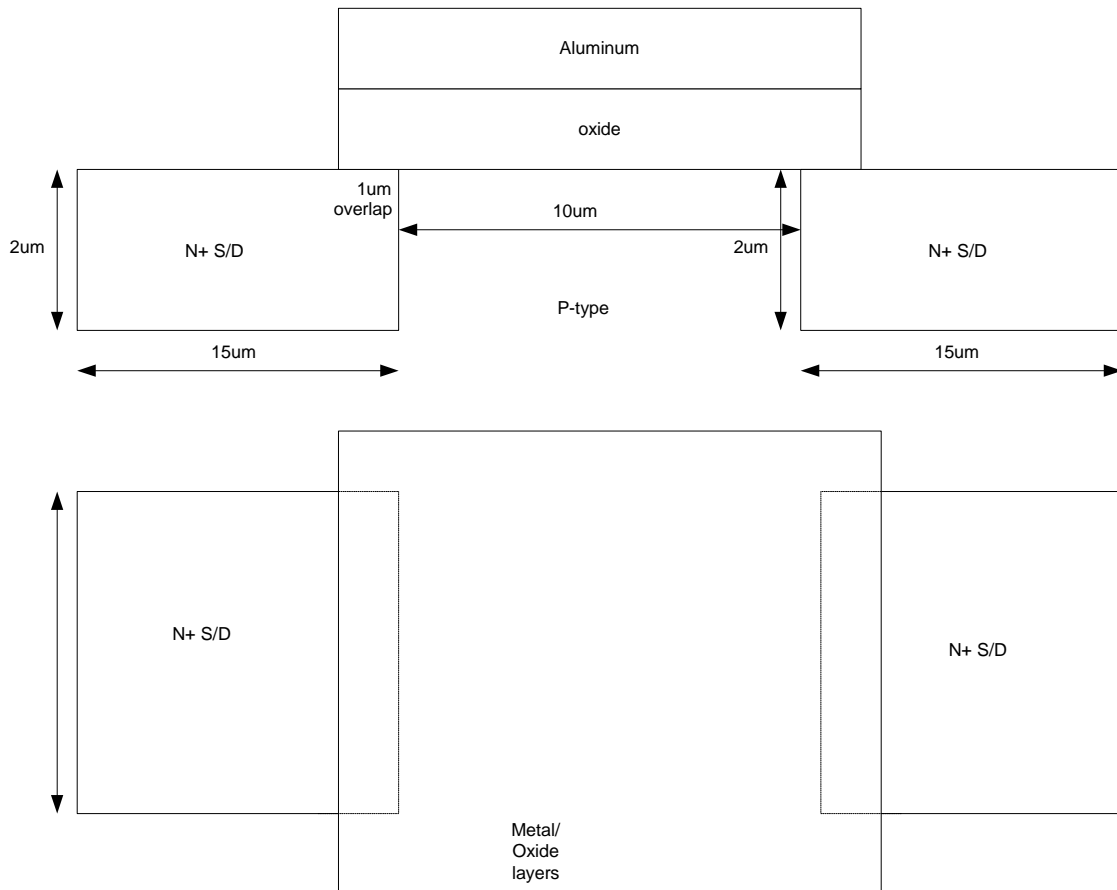


Figure 2: Top view and cross section of MOSFET.

The first thing to do is pick a bias point as the drain capacitance will depend on the drain voltage, and the output resistance depends upon the drain current. We have everything we need to calculate V_T , and we can look up the mobility (assuming the mobility is cut in half due to the electrons being confined in a channel, and that it will not change with the applied voltages).

$$q := 1.6 \cdot 10^{-19} \text{ C} \quad \epsilon_{\text{Si}} := 11.8 \cdot 8.85 \cdot 10^{-14} \frac{\text{F}}{\text{cm}} \quad k := 1.38 \cdot 10^{-23} \frac{\text{J}}{\text{K}}$$

$$T := 300 \text{ K} \quad N_A := 1 \cdot 10^{17} \text{ cm}^{-3} \quad \epsilon_{\text{oxide}} := 3.9 \cdot 8.85 \cdot 10^{-14} \frac{\text{F}}{\text{cm}}$$

$$n_i := 9.65 \cdot 10^9 \text{ cm}^{-3} \quad Q_i := 5 \cdot 10^{10} \cdot \frac{q}{\text{cm}^2} \quad Q_i = 8 \times 10^{-9} \text{ C cm}^{-2}$$

$$\phi_F := \frac{k \cdot T}{q} \cdot \ln\left(\frac{N_A}{n_i}\right) \quad \phi_{\text{ms}} := -.6 \text{ V} - \frac{k \cdot T}{q} \cdot \ln\left(\frac{N_A}{n_i}\right) \quad \phi_{\text{ms}} = -1.018 \text{ V}$$

$$Q_D := -2 \cdot \left(\epsilon_{\text{Si}} \cdot q \cdot N_A \cdot \phi_F\right)^{\frac{1}{2}} \quad Q_D = -1.671 \times 10^{-7} \frac{\text{C}}{\text{cm}^2}$$

$$d := 0.015 \cdot 10^{-4} \text{ cm} \quad C_{\text{oxide}} := \frac{\epsilon_{\text{oxide}}}{d}$$

$$V_{T0} := \phi_{\text{ms}} - \frac{Q_i}{C_{\text{oxide}}} - \frac{Q_D}{C_{\text{oxide}}} + 2 \cdot \phi_F$$

$$V_{T0} = 0.51 \text{ V}$$

$$U_T := .0259 \text{ V}$$

In the figure below you see the MOSFET in the linear and saturation mode with both channel length modulation and without. I used $L=10\mu\text{m}$ rather than L minus the depletion widths, because I am not interested in the current yet. I also need to pick a drain voltage to get the proper depletion width. If we assume a V_{DD} of 5 volts and an overdrive voltage of .3 Volts ($V_{GS}-V_T=.3\text{V}$) we see that a V_D of 2.5 volts puts us around the middle of the saturation region, thus allowing maximum swing of the output.

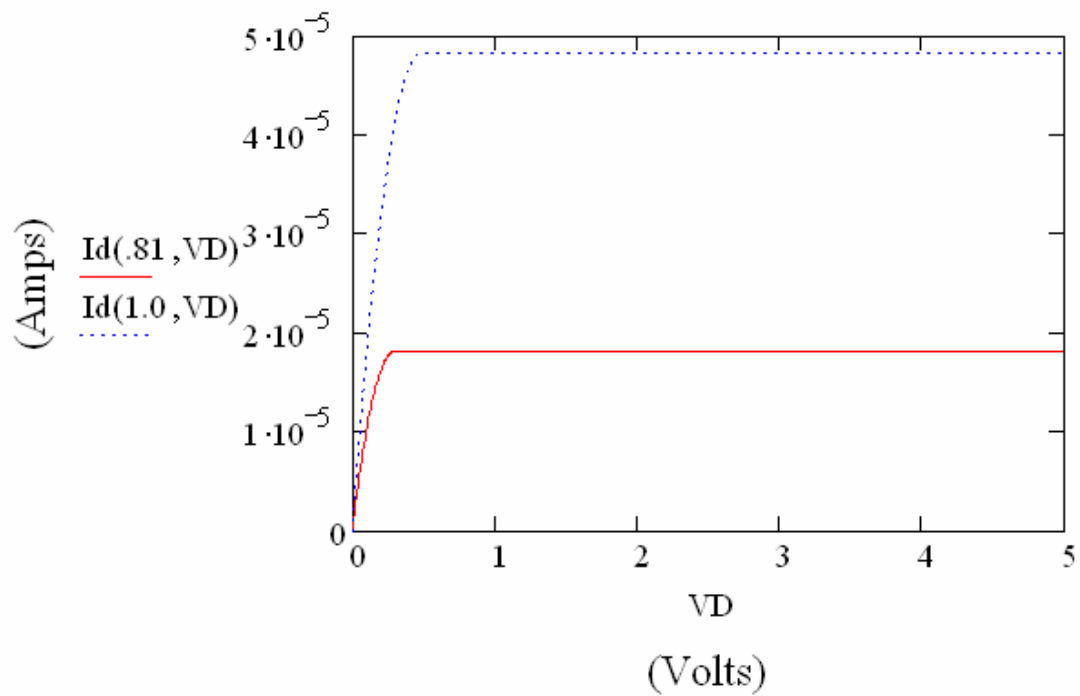


Figure 3: I_D, V_D , with V_G as a parameter, no channel width modulation.

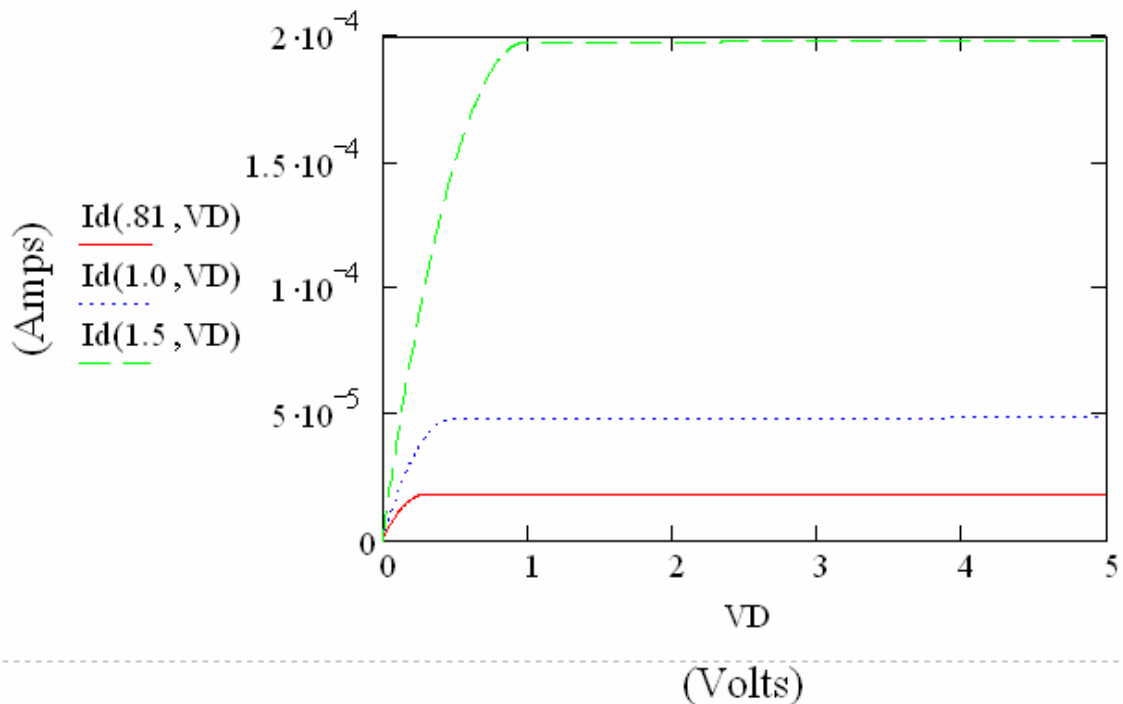


Figure 4: I_D, V_D , with V_G as a parameter, with channel width modulation.

Now we have a V_{DS} of 2.5 Volts we can calculate the depletion width and thus the channel length and the drain capacitance.

$$N_a := 10^{17} \text{ cm}^{-3}$$

$$N_d := 10^{19} \text{ cm}^{-3} \quad DD := 10 \cdot 10^{-4} \text{ cm} \quad Z := 50 \cdot 10^{-4} \text{ cm}$$

$$R_j := 2.0 \cdot 10^{-4} \text{ cm}$$

$$V_o := \frac{k \cdot T}{q} \cdot \ln \left(\frac{N_a \cdot N_d}{n_i^2} \right) \quad V_o = 0.955 \text{ V} \quad +$$

$$\text{Area_sidewall_1} := 2 \cdot DD \cdot R_j \quad \text{Area_sidewall_1} = 4 \times 10^{-7} \text{ cm}^2$$

$$\text{Area_sidewall_2} := 2 \cdot Z \cdot R_j \quad \text{Area_sidewall_2} = 2 \times 10^{-6} \text{ cm}^2$$

$$\text{Area_bottom} := Z \cdot DD \quad \text{Area_bottom} = 5 \times 10^{-6} \text{ cm}^2$$

$$\text{Area_Total} := \text{Area_sidewall_1} + \text{Area_sidewall_2} + \text{Area_bottom}$$

$$\text{Area_Total} = 7.4 \times 10^{-6} \text{ cm}^2 \quad V_d := -2.5 \text{ V} \quad \text{A positive } V_D \text{ is a negative voltage on the diode.}$$

$$W_{\text{Depletion}} := \sqrt{\frac{2 \cdot \epsilon_{\text{Si}}}{q} \cdot \frac{N_a + N_d}{N_a \cdot N_d} \cdot (V_o - V_d)} \quad W_{\text{Depletion}} = 2.134 \times 10^{-5} \text{ cm}$$

$$C_{\text{DB}} := \frac{\text{Area_Total} \cdot \epsilon_{\text{Si}}}{W_{\text{Depletion}}} \quad C_{\text{DB}} = 3.621 \times 10^{-13} \text{ F}$$

The overlap capacitance is the oxide capacitance due to the fact the mos over the drain is all ways in accumulation. This assumes that the depletion width is not a significant portion of the channel length.

$$\text{overlap} := 1 \cdot 10^{-4} \text{ cm}$$

$$C_{\text{overlap}} := C_{\text{oxide}} \cdot Z \cdot \text{overlap} \quad C_{\text{overlap}} = 1.151 \times 10^{-13} \text{ F}$$

R_{in} is considered to be infinite, and thus is not considered because the oxide thickness is 5 times the thickness at which tunnel occurs (150Å vs. 30Å).

The capacitance from the gate to drain in the triode region is $1/2C_{ox} \cdot Z \cdot L$, and is zero in the saturation region. $C_{gb}=0$.

C_{gs} in saturation is $2/3$'s th Oxide capacitance value.

$$L := 10 \cdot 10^{-4} \text{ cm}$$

$$C_{gs} := \frac{2}{3} \cdot C_{oxide} \cdot Z \cdot L \quad C_{gs} = 7.67 \times 10^{-13} \text{ F}$$

This assumes that the depletion width is not a significant portion of the channel length. Is this a good assumption?

$$V_d := -2.5 \text{ V}$$

$$W_{Depletion_Drain} := \sqrt{\frac{2 \cdot \epsilon_{Si}}{q} \cdot \frac{N_a + N_d}{N_a \cdot N_d} \cdot (V_o - V_d)}$$

$$W_{Depletion_Drain} = 2.134 \times 10^{-5} \text{ cm}$$

$$V_d := 0 \text{ V}$$

$$W_{Depletion_Source} := \sqrt{\frac{2 \cdot \epsilon_{Si}}{q} \cdot \frac{N_a + N_d}{N_a \cdot N_d} \cdot (V_o - V_d)}$$

$$W_{Depletion_Source} = 1.122 \times 10^{-5} \text{ cm}$$

$$L_{eff} := L - W_{Depletion_Source} - W_{Depletion_Drain}$$

$$L_{eff} = 9.674 \times 10^{-4} \text{ cm}$$

$$\frac{L_{eff}}{L} = 0.967 \quad +$$

It looks like it is a good assumption.

G_m is given by:

$$V_G := V_{T0} + .3V$$

$$\mu_n := 350 \frac{\text{cm}^2}{\text{V}\cdot\text{s}}$$

$$g_m := \frac{Z \cdot \mu_n \cdot C_{\text{oxide}}}{L} \cdot (V_G - V_{T0}) \quad g_m = 1.208 \times 10^{-4} \frac{1}{\Omega}$$

To find R_o we can assume that channel length modulation does not affect the hand calculations much by looking at figures 3 and 4. This is not all ways the case. We just need a simple long channel equation for the saturation current.

$$I_D := \frac{Z}{2 \cdot L} \cdot \mu_n \cdot C_{\text{oxide}} \cdot (V_G - V_{T0})^2 \quad I_D = 1.812 \times 10^{-5} \text{ A}$$

$$R_o := \frac{1}{\lambda \cdot I_D} \quad R_o = 5.519 \times 10^7 \Omega \quad +$$

The final thing to calculate is the resistance of the gate, drain and source. The lead resistance is the same for all three terminals.

The resistance to the drain and source has to include the contact resistance and the resistance of the source and drain.

We can make a worst case that the Width will be Z , the Length will be DD , and the height is R_j .

$$R_{\text{Contact}} := 10\Omega \quad \text{Resistivity} := 8 \cdot 10^{-3} \Omega \cdot \text{cm} \quad \text{From fig 7 page 55 of text.}$$

$$R_j := 2 \cdot 10^{-4} \text{ cm} \quad R_{\text{Diff}} := \frac{DD}{Z \cdot R_j} \cdot \text{Resistivity} \quad R_{\text{Diff}} = 8\Omega$$

$$R_{\text{Drain}} := R_{\text{Lead}} + R_{\text{Contact}} + R_{\text{Diff}} \quad R_{\text{Drain}} = 18.15\Omega$$

$$R_{\text{Source}} := R_{\text{Drain}}$$

#2 There are two limits on the maximum drain voltage. (The minimum drain voltage is $V_G - V_T$, so that the device will be in saturation). The break down voltage of the drain pn junction and when the depletion widths touch (punch through or DIBL) is another method by which we lose control of the gate. We need to find when L_{eff} equals zero, or we can look up the breakdown voltage from figure 22 page 129. With the substrate doping we get a breakdown voltage of 10Volts, see if we still have a channel at that voltage.

$$V_d := -10V$$

$$W_{\text{Depletion}} := \sqrt{\frac{2 \cdot \epsilon_{\text{Si}}}{q} \cdot \frac{N_a + N_d}{N_a \cdot N_d} \cdot (V_o - V_d)} \quad W_{\text{Depletion}} = 3.8 \times 10^{-5} \text{ cm}$$

$$W_{\text{Source}} := \sqrt{\frac{2 \cdot \epsilon_{\text{Si}}}{q} \cdot \frac{N_a + N_d}{N_a \cdot N_d} \cdot (V_o - 0)}$$

$$W_{\text{Drain}} := \sqrt{\frac{2 \cdot \epsilon_{\text{Si}}}{q} \cdot \frac{N_a + N_d}{N_a \cdot N_d} \cdot (V_o - V_d)}$$

$$L_{\text{eff}} := L - W_{\text{Source}} - W_{\text{Drain}}$$

$$L_{\text{eff}} = 9.508 \times 10^{-4} \text{ cm} \quad \text{We still have a channel, so the limit is the breakdown voltage.}$$

#3

$$V_{\text{FB}} := \phi_{\text{ms}} - \frac{Q_i}{C_{\text{oxide}}} \quad V_{\text{FB}} = -1.053 \text{ V}$$

$$\Psi_{\text{B}} := \phi_{\text{F}} \quad C_o := C_{\text{oxide}} \quad V_{\text{BS}} := 1 \text{ V}$$

$$V_{\text{T}} := V_{\text{FB}} + 2 \cdot \Psi_{\text{B}} + \frac{\sqrt{2 \cdot \epsilon_{\text{Si}} \cdot q \cdot N_A \cdot (2 \cdot \Psi_{\text{B}} + V_{\text{BS}})}}{C_o}$$

$$V_{\text{T}} = 0.86 \text{ V}$$

#4

$$\Delta V_T := \frac{-q \cdot N_A \cdot W_m \cdot R_j}{C_o \cdot L} \cdot \left(\sqrt{1 + \frac{2 \cdot W_m}{R_j}} - 1 \right)$$

$$\Delta V_T = -7.399 \times 10^{-3} \text{ V}$$

$$V_T := V_{T0} - \Delta V_T \quad V_T = 0.517 \text{ V} \quad +$$

We do not see a significant different in V_T .

#5

DIBL causes a large current to be present when there is a large v_{ds} . In the case of the CMOS inverter when v_{in} is zero volts, the pmos is on and the nmos is supposed to be off, but it leaks. When v_{in} is high the nmos is on and the pmos is supposed to be off but it leaks. The result is that the inverter can not drive the voltage to vdd or ground.

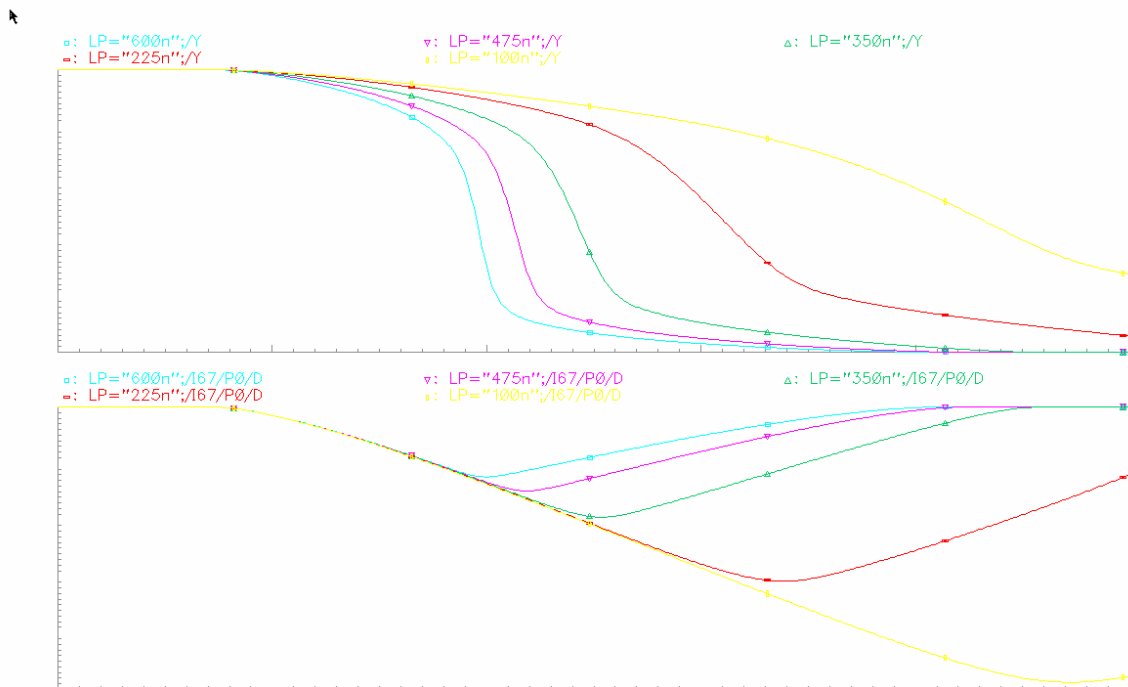


Figure 5: V_{out} vs. V_{in} , CMOS inverter with LP showing DIBL.

The upper plot is v_{out} vs v_{in} , and the lower is the drain current. We see that for a small channel length of the pmos (yellow) the current never goes to zero, and v_{out} never goes to zero.