

Week 9: Fermi-Dirac Statistics Announcements

Boltzmann Probability Function

- Electrons are going to move around energy levels as they collide/ interact with one another
- Assume a particle like view of electrons: two electrons at E_1 and E_2 interact to result in electrons at E_3 and E_4

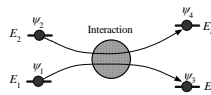


Fig. 4.23: Two electrons initially with wavefunctions ψ_1 and ψ_2 at E_1 and E_2 interact and end up at different energies at E_3 and E_4 . Their corresponding wavefunctions are ψ_3 and ψ_4 .

Boltzmann Probability Function

- In steady-state (no net motion of electrons)
 - the probability of going back the other way (E_3 and E_4 interact to result in E_1 and E_2) must be just as likely
- Also: energy is conserved:

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Boltzmann Statistics

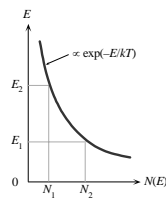


Fig. 4.24: The Boltzmann energy distribution describes the statistics of particles, e.g. electrons, when the particles do not interact with each other, i.e. when there are very few electrons compared with the number of available states.

- Solution to these 2 equations
 $P(E)=f(E)=A\exp(-E/kT)$
- Number of particles at E_1 and E_2 are N_1 and N_2
- Their ratio depends on the energy spread

$$\frac{N_2}{N_1} = \exp\left(-\frac{E_2 - E_1}{kT}\right)$$

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Assumptions in Boltzmann Statistics

- Boltzmann probability function ignores Pauli's exclusion principle
- The number of states at a given energy just increases exponentially with the assumption that any number of particles can have a particular energy
- This is an okay assumption for cases where there are a limited number of electrons and lots of states (so the odds are you won't have more than two in the same state)

Fermi-Dirac Probability Function

- In general, though, we need to factor in Pauli's Exclusion principle
- For an electron to go from E_1 and E_2 to E_3 and E_4
 - we have to GUARANTEE that E_3 and E_4 are empty!

Fermi Dirac Function

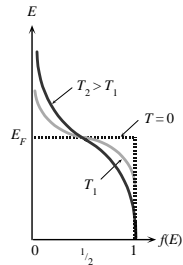


Fig. 4.25: The Fermi-Dirac function, $f(E)$, describes the statistics of electrons in a solid. The electrons interact with each other and the environment so that they obey the Pauli Exclusion Principle.

■ New solution:

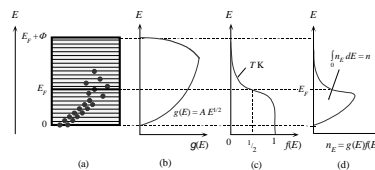
$$f(E) = \frac{1}{1 + \exp\left(\frac{E - E_F}{kT}\right)}$$

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Concentration of Electrons (n)



$$n = \int_0^{\text{top}} n_e dE = \int_0^{\text{top}} g(E)f(E)dE$$

Fig. 4.26: (a) Above 0 K, due to thermal excitation, some of the electrons are at energies above E_F . (b) The density of states, $g(E)$ vs E in the band. (c) The probability of occupancy of a state at an energy E is $f(E)$. The product $g(E)f(E)$ is the number of electrons per unit energy per unit volume or electron concentration per unit energy. The area under the curve with the energy axis is the concentration of electrons in the band.

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Solving for n at any E in the band

$$n = \int_0^{\text{top}} n_e dE = \int_0^{\text{top}} g(E)f(E)dE$$

End up with:

Plug in for f(E) and g(E)

$$f(E) = \frac{1}{1 + \exp\left(\frac{E - E_F}{kT}\right)}$$

$$g(E) = 8\pi\sqrt{2}\left(\frac{m_e}{h^2}\right)^{3/2} \sqrt{E}$$

$$n = \frac{8\pi\sqrt{2}m_e^{3/2}}{h^3} \int_0^{\infty} \frac{E^{1/2}dE}{1 + \exp\left(\frac{E - E_F}{kT}\right)}$$

$$n(E) = \frac{8\pi\sqrt{2}m_e^{3/2}}{h^3} \frac{E^{1/2}}{1 + \exp\left(\frac{E - E_F}{kT}\right)}$$

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Can Solve for E_F for Metals

- For metals, if you integrate $n(E)$ over the valence band you should get the n we have been calculating before (density of free electrons)
- If you know n , you can calculate E_F for a metal

$$E_{F0} = \left(\frac{h^2}{8m_e}\right)\left(\frac{3n}{\pi}\right)^{2/3}$$

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Fermi Energy Changes With T

- The Fermi energy is defined as:

- As you increase the temperature, the Fermi energy will move
 - Because electrons get excited to higher states

$$E_F(T) = E_{F0} \left[1 - \frac{\pi^2}{12} \left(\frac{kT}{E_{F0}} \right)^2 \right]$$