Week 4: Particles & Waves
Announcements

Particles vs Waves

- We have so far talked about electrons being accelerated and scattered.
  - These are particle-like properties and can explain basic electrical phenomena such as current and conductivity.
- However, electrons can also exhibit wave-like properties.
  - Certain electrical phenomena can only be explained using the wave-like properties of electrons.
- No phenomena is known that needs both the particle and wave natures to be explained
- Electrons have a wave/ particle dual nature.
- Light also has both a wave-like and particle like nature.

Light as a Wave

- Light can be expressed as a wave.
- Light can constructively or destructively interfere with other waves.
- Constructive interference
- Destructive interference

Review of Wave Definitions

- ν
- $\nu = \lambda \upsilon$
  - only for EM waves, not other waves
  - for an EM wave traveling in vacuum (or air) $v = c = 2.998\times10^{8}$m/s

$$v = \frac{\omega}{k} \qquad k = \frac{2\pi}{\lambda} \qquad \omega = 2\pi\upsilon$$

Electromagnetic Waves

- Light can be expressed as perpendicular **electromagnetic waves**
- $E_y(x,t) = E_{yo} \sin (kx - \omega t)$
- There is also a similar expression for the magnetic component

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Electromagnetic Wave

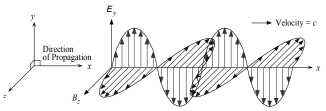

Fig. 3.1: The classical view of light as an electromagnetic wave. An electromagnetic wave is a travelling wave which has time varying electric and magnetic fields which are perpendicular to each other and to the direction of propagation.

From *Principles of Electronic Materials and Devices, Second Edition*, S.O. Kasap (© McGraw-Hill, 2002) http://Materials.Usask.Ca

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Intensity of Light

- Based on a wave definition of light, intensity is related to the velocity and the magnitude of electric field

$$I_{instanteous} = c\varepsilon_o E^2$$

$$I_{av} = \frac{1}{2} c\varepsilon_o E_o^2$$

Young's Double Slit Experiment

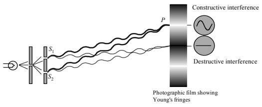

Fig.3.2: Schematic illustration of Young's double slit experiment.

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Proof of Wave Like Nature of Electrons & Light

- Light and electrons behave the same in a double slit experiment
- If light or electrons were solely particles they would not “make it through” the second slit and they would not interfere
- Diffraction is seen as a similar phenomena where the spacing between atoms in a lattice act as the slits
  - Diffraction of electrons or light is used to measure properties of the lattice (atomic spacing, crystal structure…)

Light as a Particle: The Photoelectric Effect

- Photoelectric effect
  - one of the earliest experiments that cast doubt on classical physics and gave birth to quantum physics
- The experiment proves light has a particle like nature to it by showing that light transfers energy to electrons in a billiard ball (classical physics) type way

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Description of the Experiment

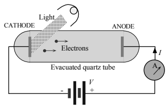

Fig. 3.4: The Photoelectric Effect.

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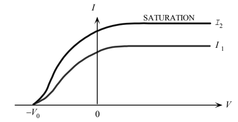
Photoelectric Effect

- Light shines on the target (cathode)
- Electrons in the target material receive energy from the light and leave the target
- The electrons are accelerated towards the anode where they are counted by the ammeter

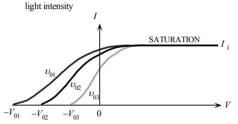
Stopping Voltage

- A negative voltage (to the anode) is used to determine the kinetic energy of the electrons
  - how much repulsive force must be exerted to prevent the electrons from reaching the anode
  - This is known as the stopping voltage (V_o)

Saturation Current



(a) Photoelectric current vs. voltage when the cathode is illuminated with light of identical wavelength but different intensities (*I*). The saturation current is proportional to the light intensity



(b) The stopping voltages and therefore the maximum kinetic energies of the emitted electrons depend on the frequency of light, υ.

13

Fig. 3.5: Results from the photoelectric experiment.

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Results

- The current (number of electrons emitted saturates)
- The saturation current increases with increasing intensity
- The saturation current does not depend on frequency, only on intensity
- The stopping voltage does not depend on the intensity, only the frequency

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Cut-off Frequency

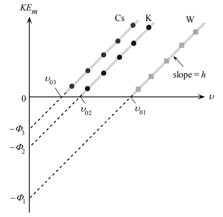

Fig. 3.6: The effect of varying the frequency of light and the cathode material in the photoelectric experiment. The lines for the different materials have the same slope of *h* but different intercepts.

14

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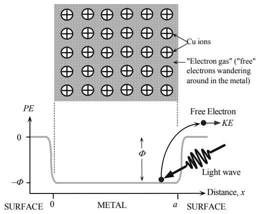
Results

- The KE of emitted electrons depends on the properties of the frequency of light

- There is a threshold frequency of light, below which we get no photoelectric effect.
- The threshold frequency is a materials' property

Work Function



$$\Phi = h\upsilon_o$$

Fig. 3.7: The PE of an electron inside the metal is lower than outside by an energy called the workfunction of the metal. Work must be done to remove the electron from the metal.

15

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Discrepancies with Results

- The results of the photoelectric effect are contradictory to what we expect based on the wave view of light
- The number of emitted electrons (current) depends of intensity but the energy of emitted electrons (V_o) doesn't
- As the intensity increases, the magnitude of the electric field increases
  - The increase in this force (electric field) should result in more energetic electrons

Einstein's Explanation

- When a_______ hits an electron- all of that energy is transferred to the electron.
- The ________ all have the same energy though (for a given frequency of light)
- Intensity of light must represent how many ________ are present

Particle-like Definition of Intensity

- Particle definition of intensity:

$$I = \Gamma_{ph} h\upsilon$$

$$\Gamma_{ph} = \frac{\Delta N_{ph}}{A\Delta t}$$

- Increasing I increase N_{ph}
  - which results in more emitted electrons and a higher saturation current
- The energy of the wave is the same as the energy of any individual photon in that wave
  - The energy relates to the frequency but not the intensity.

Compton Scattering

- Compton scattering is another particle-like phenomena of light
- The light loses energy (transfers it to the electron)
  - Because momentum must be conserved- the light must have momentum

Compton Scattering

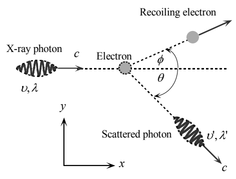

Fig. 3.9: Scattering of an x-ray photon by a "free" electron in a conductor.

17

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Momentum Equations

- Photons are mass-less particles that contain momentum
- Compton experiment showed:

$$p = \frac{h}{\lambda} = \hbar k \qquad \hbar = \frac{h}{2\pi}$$

- This is known as the
  - It applies to anything (light, electrons…)
  - Allows us to transition between particle & wave equations

Wave Like Nature of Electrons

- Electrons act just like light in Young's double slit and diffraction experiments.
  - This shows that electrons, like light, have wave-like properties
- Peaks and valleys in light wave
  - represents highs and lows in electric and magnetic fields
- Peaks and valleys in electron wave
  - represents highs and lows in the probability of finding an electron at that spot in time or space

Wave Function of An Electron

- Max Born made an equation for the wave like nature of electrons which was similar to that for light
- Remember the wavefunction of electric field
  - $E(x,t)=E_o \sin(kx-\omega t)$
- Born defined Ψ **as the wavefunction of an electron**
  - $|\Psi^2|$ as the probability of finding an electron at a spot in 1D (or volume in 3D)
  - Ψ itself has very little physical meaning (and it can be imaginary)
  - $|\Psi^2|$ must be real; it is actually $\Psi^* \Psi$

Wave Function Equations

- Total wave function

$$\Psi(x,t) = \Psi(x)\exp\left(-\frac{jEt}{\hbar}\right) = \Psi(x)\exp(-j\omega t)$$

- Time independent Schrodinger's Equation

$$\frac{d^2\Psi}{dx^2} + \frac{2m}{\hbar^2}(E-V)\Psi = 0$$

Uses of Schrodinger's Equation

- This equation has been found to predict every wavelike phenomena for electrons
- Based on experiment not theory
- Used to describe the wave like nature of electrons.

- Where we are going: Schrodinger's equation is what is used to tell us the allowable energy states of an electron.

What's Need to Solve for Ψ

- Boundary conditions to solve for Ψ
  - Ψ must be continuous and d Ψ/dx must be continuous
- Ψ must be single valued
  - This is because Ψ^2 is the probability of finding an electron
  - Two spots can't have equal probability of the electron being there

Required Boundary Conditions

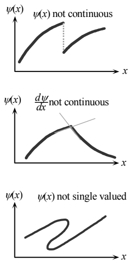

Fig. 3.14: Unacceptable forms of $\psi(x)$

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Free Electron (Solution 1)

- One condition to simplify and solve Schrodinger's equation is to assume V=0
  - There is no voltage force acting on the electron
  - The electron is free

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{2m}{\hbar^2}(E)\psi = 0$$

- A solution to this differential equation is a complex exponential function: $\psi(x)=Ae^{ikx}$
  - Remember the Euler equations tell us also that $\psi(x)=Ae^{ikx}=A(\cos kx + i\sin(kx))$
  - where $k=2\pi/\lambda$ and A is the normalizing constant

Solving for E

- Start with: $\psi(x)=Ae^{ikx}$

$$\frac{\partial \psi}{\partial x} = iAke^{ikx}$$

$$\frac{\partial^2 \psi}{\partial x^2} = -k^2 Ae^{ikx}$$

$$-k^2 Ae^{ikx} + \frac{2mE}{\hbar^2} Ae^{ikx} = 0$$

$$k^2 = \frac{2mE}{\hbar^2}$$

$$E = \frac{\hbar^2 k^2}{2m}$$

Dispersion Relationship

- So for a free electron, the S.W.E. gives us the *dispersion relation*
  - the relationship between *energy* and *wave number*

- E is parabolic function of k