Week 3: Metal Resistivity Announcements

Review of Last Time's Equations

- **What is the drift velocity?**
- What is the drift mobility?
- What is conductivity in terms of drift mobility?

Temperature Dependence of Resistivity of Metals

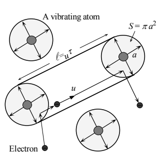

Fig. 2.4: Scattering of an electron from the thermal vibrations of the atoms. The electron travels a mean distance $\ell = u\tau$ between collisions. Since the scattering cross sectional area is S, in the volume $S\ell$ there must be at least one scatterer, $N_s(Su\tau) = 1$.

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T Dependence of Vibrations

- T is proportional to cross sectional area atoms takes up when vibrating (πa^2)
- a^2 can be related to temperature using a spring analogy (temperature adds energy which causes the atom to vibrate).

$$\frac{1}{4}Ma^2\omega^2 = \frac{1}{2}kT$$

- M is the mass of the atom, ω is the vibrational frequency

The Math of the T Dependence

- How does scattering depend on temperature?

$$\tau \propto \frac{1}{\pi a^2} \propto \frac{1}{T} = \frac{C}{T}$$

- How does resistivity depend on temperature?

$$\rho_T = \frac{1}{\sigma_T} = \frac{1}{en\mu_D} = \frac{m_e T}{e^2 nC} = AT$$

Electron Scattering Events

- We discussed how electrical resistance comes about due to the scattering of electrons by the lattice (thermal scattering)
- Other scattering events also exist. Some examples are:

Mathiessen's Rule

- All these scattering events are accounted for by Mathiessen's Rule

$$\frac{1}{\tau} = \frac{1}{\tau_L} + \frac{1}{\tau_I}$$

$$\frac{1}{\mu} = \frac{1}{\mu_L} + \frac{1}{\mu_I}$$

Overall Temperature Dependence of Resistivity

- Unlike scattering due to the lattice, residual resistivity has very little temperature dependence
  - This gives:
- Typically rather than list A & B, they give the TCR (temperature coefficient of resistivity) α_o

$$\rho = \rho_o\left[1 + \alpha_o\left(T - T_o\right)\right] \qquad \alpha_o = \frac{1}{\rho_o}\left[\frac{\delta\rho}{\delta T}\right]$$

Why is the T Dependence of Iconel, Monel, and NiCr Different Than the Elements??

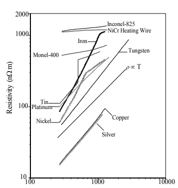

Fig. 2.6: The resistivity of various metals as a function of temperature above 0 °C. Tin melts at 505 K whereas nickel and iron go through a magnetic to non-magnetic (Curie) transformations at about 627 K and 1043 K respectively. The theoretical behavior ($\rho \sim T$) is shown for reference.
[Data selectively extracted from various sources including sections in *Metals Handbook*, 10th Edition, Volumes 2 and 3 (ASM, Metals Park, Ohio, 1991)]

Effect of Various Scattering Mechanisms on Resistivity

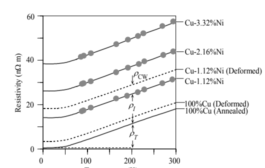

Fig. 2.8: Typical temperature dependence of the resistivity of annealed and cold worked (deformed) copper containing various amount of Ni in atomic percentage (data adapted from J.O. Linde, *Ann. Pkysik*, **5**, 219 (1932)).

Resistivity Increases with Alloying

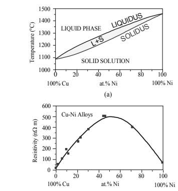

Fig. 2.10(a) Phase diagram of the Cu-Ni alloy system. Above the liquidus line only the liquid phase exists. In the *L* + *S* region, the liquid (*L*) and solid (*S*) phases coexist whereas below the solidus line, only the solid phase (a solid solution) exists. (b) The resistivity of the Cu-Ni alloy as a function of Ni content (at.%) at room temperature. [Data extracted from Metals Handbook-10th Edition, Vols 2 and 3, ASM, Metals Park, Ohio, 1991 and Constitution of Binary Alloys, M. Hansen and K. Anderko, McGraw-Hill, New York, 1958]

Nordheim's Rule

- Nordheim's Rule relates resistivity to alloy (impurity) level
- This applies to solid solutions
  - two materials completely miscible (one phase)

$$\rho_{Impurity} = C\,X\,(1 - X)$$

What is happening in the Cu-Au system?

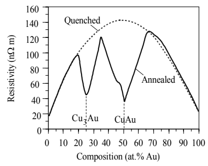

Fig. 2.11: Electrical resistivity vs. composition at room temperature in Cu-Au alloys. The quenched sample (dashed curve) is obtained by quenching the liquid and has the Cu and Au atoms randomly mixed. The resistivity obeys the Nordheim rule. On the other hand, when the quenched sample is annealed or the liquid slowly cooled (solid curve), certain compositions (Cu_3Au and CuAu) result in an ordered crystalline structure in which Cu and Au atoms are positioned in an ordered fashion in the crystal and the scattering effect is reduced.