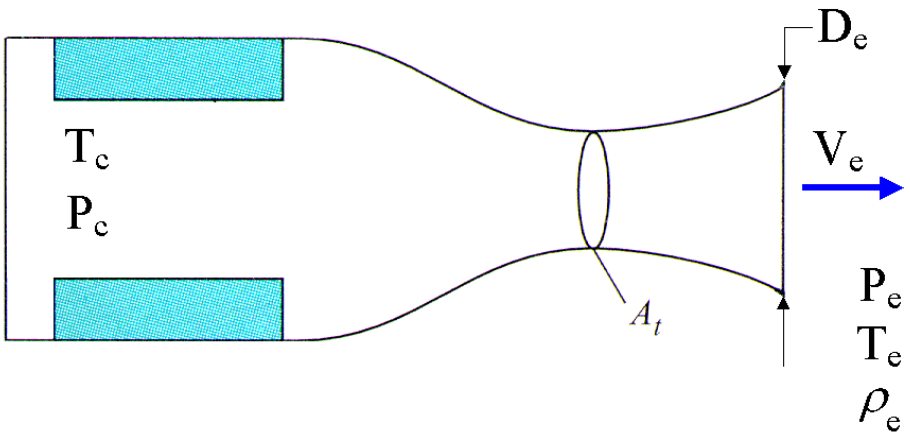
CONTINUITY

Example Problem: Flow through a Solid-Propellant Rocket-Engine

Determine the rate at which propellant is being consumed in the combustion chamber of the solid-propellant rocket engine shown in the figure.



Given:

Combustion chamber conditions:

Pressure	p_c = 950 kPa
Temperature	T_c = 1500 K

Nozzle exit conditions:

Pressure	p_e = 90 kPa
Temperature	T_e = 750 K
Exhaust velocity	V_e = 1,150 m/sec
Molecular weight of exhaust gas	M = 31
Exit diameter	D_e = 18 cm

Solution:

Let's start with the simplest and most general mathematical expression for continuity:

$$\frac{d(m_{cv})}{dt} = \dot{m}_{in} - \dot{m}_{out} \qquad (1)$$

Rocket engines don't have an inlet so $\dot{m}_{in} = 0$ (2)

$$\dot{m}_{out} = r_e A_e V_e \tag{3}$$

where $$r_e = \frac{p_e}{RT_e} = \frac{90{,}000}{268(750)} = 0.447\ kg/m^3 \tag{4}$$

The value of the gas constant was calculated from:

$$R = \frac{R_{universal}}{M} = \frac{8{,}313}{31} = 268\ \ J/kg.K$$

Using eqs.(2-4) in eq.(1):

$$\frac{d(m_{cv})}{dt} = \frac{d(m_{propellant})}{dt} = -\dot{m}_{out} = -0.447\left[\frac{p\ (0.18)^2}{4}\right]1{,}150 \Rightarrow \frac{d(m_{propellant})}{dt} = -13.1\ kg/\sec$$

The minus sign indicates that the propellant mass inside the combustion chamber is decreasing.