HYDROSTATICS

Example Problem: ***Hydrostatic Forces on Submerged Surfaces***
Pressurized Tank

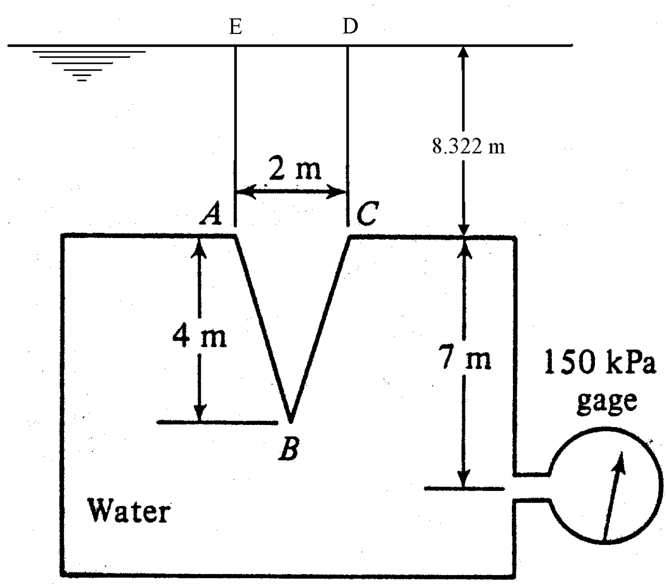
The tank shown in the figure is filled with pressurized water. Calculate the net hydrostatic force on the conical surface ABC.

Given:

Temperature of water:	20^0C
Height of cone:	4 m
Base diameter of cone:	2 m
Pressure measured at a depth of 7 m from the top of the tank:	150 kPa-gage

Solution

Pascal's principle applies here; the extra pressure in the tank will be felt equally everywhere, including the surface of the cone. There are two ways to approach this problem.



Method 1

If the tank was not pressurized, at 7 m depth the pressure would be:

$$p = g_{H_2O} h = 9{,}790(7) = 68{,}530\,Pa = 68.53\,KPa$$

However, the pressure gage at 7 m now measures 150 Kpa-gage. Ask the question: At what depth would I get the same pressure in an open tank? I can answer this question using the hydrostatic eq. again:

$$h_{equiv.} = \frac{p}{g_{H_2O}} = \frac{150{,}000}{9{,}790} = 15.322 \ m$$

In other words, the hydrostatic pressure distribution on the cone will be the same as if its base were submerged at a depth of 15.322 – 7 = 8.322 m.

The pressure distribution on the surface of the cone is symmetrical, so the net horizontal force on it is zero (F_H=0). Therefore, the net hydrostatic force on the cone will be vertical, pushing the cone up since the water is underneath the cone surface.

$$F_V = W_{H_2O-above} = g_{H_2O}\left[\forall_{ABC} + \forall_{ACDE}\right] = 9{,}790\left[\frac{1}{3}p(1)^2 4 + \frac{p}{4}2^2(8.322)\right] \Rightarrow F_{V,Cone} = 297\,kN$$

Method 2

We can calculate the force on the cone as if the tank were not pressurized. Then, we can add the additional force due to the extra pressure in the tank. Pretending the base of the cone is at the free surface of the water;

$$F_{V1} = W_{H_2O-above} = g_{H_2O}\forall_{ABC} = 9{,}790\left(\frac{1}{3}p(1)^2 4\right) = 41{,}008\,kN \cong 41\,kN$$

Now the additional pressure, equally distributed over the surface of the cone is:

$$\Delta p = g_{H_2O}\Delta h = 9{,}790(8.322) = 81{,}472\,Pa$$

and the additional vertical force due to this pressure is:

$$F_{V2} = \Delta p A_{base-cone} = 81{,}472\left[p(1)^2\right] = 255{,}953\,N \cong 256\,kN$$

So, the total hydrostatic force on the cone is:

$_{V,Cone}$ $_{V1}$ $_{V2}$ $_{V}$