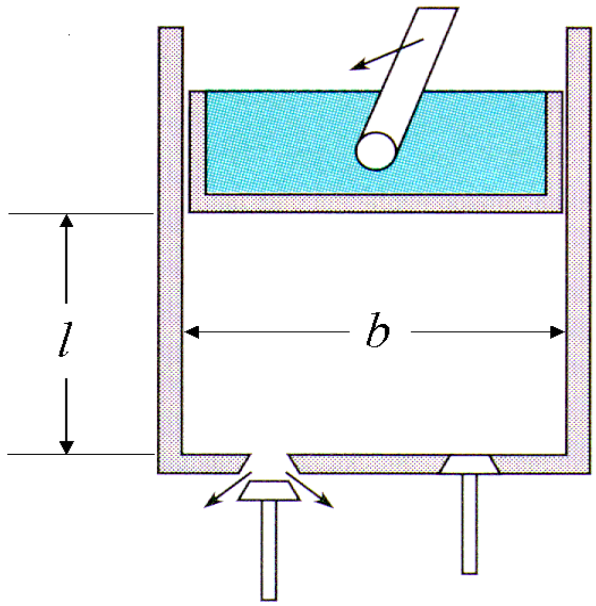
CONTINUITY

Example Problem: Flow through the Cylinder of a Piston-Engine



A piston is moving up during the exhaust stroke of a 4-cycle engine. Determine the rate at which the density of the mixture is changing in the cylinder.

Given:

Bore of the cylinder	b = 10 cm
Distance between piston and head	l = 10 cm
Valve opening area	A_v = 1 cm^2
Piston velocity	V_p = 30 m/s
Chamber pressure	p_c = 300 kPa
Chamber temperature	T_c = 600 ^{0}C
Gas constant for mixture in cylinder	R = 350 J/kg.K

Rate of mass outflow through the exhaust port: $\dot{m} = 0.65\frac{p_c A_v}{\sqrt{RT_c}}$

Assumptions:

1. The density (ρ_c) and the pressure (p_c) are uniform inside the cylinder. Strictly speaking, this may not be true at any moment of the exhaust stroke, or any other stroke for that matter, as there are always pockets of higher / lower density inside the cylinder. However, without this assumption the problem would be too complicated to solve.
2. The gas is ideal (so we can use the ideal gas law).

<u>Solution:</u>

Let's start with the simplest and most general mathematical expression for continuity:

$$\frac{d(m_{cv})}{dt} = \dot{m}_{in} - \dot{m}_{out} \qquad (1)$$

Since we are concerned with the exhaust stroke, $\dot{m}_{in} = 0$ (intake valve closed) while the rate of mass outflow is given as:

$$\dot{m}_{out} = 0.65 \frac{p_c A_v}{\sqrt{RT_c}} \qquad (2)$$

Only one of the two terms is non zero on the right-hand side of eq.(1) therefore, the left-hand side term must be non zero, which means that the flow is unsteady (in other words, flow parameters change with time). We can express the mass inside the cylinder (our control volume in this problem) as:

$$m_{cv} = r_c \forall_c \qquad (3)$$

where $$A_c = \frac{p\, b^2}{4} = \frac{p\,(0.1)^2}{4} = 7.854\, x 10^{-3} m^3 \qquad (4)$$

and $$\forall_c = A_c l = 7.854\, x 10^{-3} (0.1) = 7.854\, x 10^{-4} m^3 \qquad (5)$$

Taking the derivative with respect to time:

$$\frac{d(m_{cv})}{dt} = \forall_c \frac{d r_c}{dt} + r_c \frac{d\forall_c}{dt} \qquad (5)$$

where $$\frac{d\forall_c}{dt} = A_c \frac{dl}{dt} = A_c V_p = 7.854\, x 10^{-3} (-30) = -0.2356\, m^3/\sec \qquad (6)$$

The minus sign in the velocity is used because the piston moves so that the volume of the cylinder is decreased. To find the instantaneous density inside the cylinder for the conditions given, we use the ideal gas law:

$$r_c = \frac{p_c}{RT_c} = \frac{300{,}000}{350(600+273)} = 0.982\ kg/m^3 \qquad (7)$$

Combining eqs.(2-7) with eq.(1) we get:

$$\frac{dr_c}{dt} = -\frac{r_c}{\forall_c}\frac{d\forall_c}{dt} - \frac{0.65}{\forall_c}\frac{p_c A_v}{\sqrt{RT_c}} \Rightarrow \frac{dr_c}{dt} = -249.7\frac{kg}{m^3\,\text{sec}}$$

Again, the minus sign signifies that the density inside the cylinder is decreasing, something to be expected during the exhaust stroke.