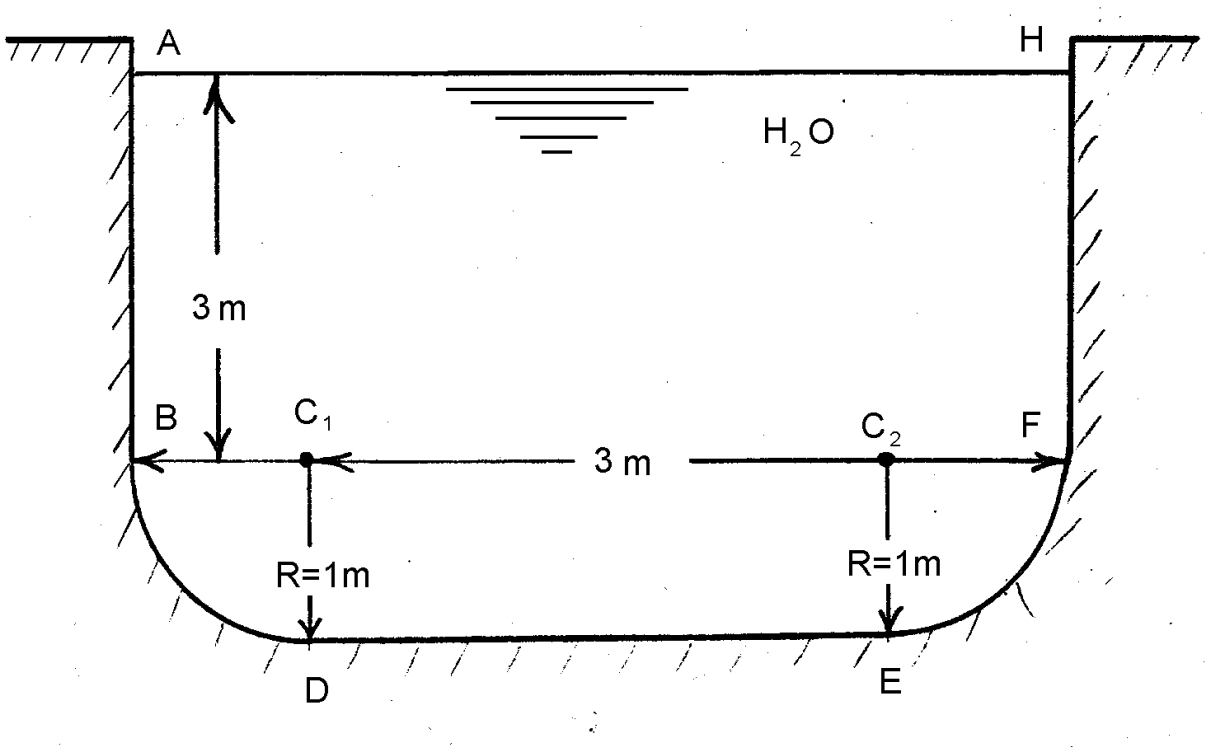
HYDROSTATICS

Previous Exam Problem: ***Hydrostatic Forces on Submerged Surfaces***
Open Conduit

Determine the magnitude, line of action, and direction of the hydrostatic forces on the open conduit sections AB, BD, DE, EF, HF. The length of the conduit is 1 km.



Solution:

Horizontal Bottom Surface (DE)

$$F_{DE} = W_{H_2O-above} = g\,(depth)(DE)L = 9{,}810(4)(3)1{,}000 \Rightarrow F_{DE} = 117.72\,MN$$

The line of action of this force will pass vertically through the center line of the conduit.

Vertical Sides (AB) and (HF)

$$F_{AB} = p_{1.5}A_{AB} = g\,(1.5)(3x1{,}000) \Rightarrow F_{AB} = 44.145\,MN = F_{HF}$$

The line of action of these forces will pass horizontally (i.e., perpendicularly to each surface) at the $\frac{2}{3}(AB) = 2\,m$ depth line.

Circular Arcs (BD) and (EF)

The horizontal component of the hydrostatic force is:

$$F_{BD-h} = p_{3.5} A_{C_1 D} = g\,(3.5)(1x1{,}000) \Rightarrow F_{BD-h} = 34.335\ \ MN = F_{EF-h}$$

The vertical component of the hydrostatic force is:

$$F_{BD-v} = W_{H_2O-above} = g\left(\forall_{BDC_1} + \forall_{ABC_1G}\right) = 9{,}810\left(\frac{p\,1^2}{4} + 3x1\right) \Rightarrow F_{BD-v} = 37.135\,MN = F_{EF-v}$$

The magnitude of the total hydrostatic force on the circular arcs is:

$$F_{tot} = \sqrt{F_h^2 + F_v^2} = \sqrt{34.335^2 + 37.135^2} \Rightarrow F_{tot} = 50.58\,MN$$

The distributed pressure forces on each circular arc act normal to the surface. Their line of action, therefore, passes through the center of the arc. Since the resultant hydrostatic force is the integral of these pressure forces, its line of action must also pass through the center of the arc (C_1 for BD and C_2 for EF). The direction of these forces can be defined by the angle ϕ they make with the horizontal:

$$j = \arctan\frac{F_{BD-v}}{F_{BD-h}} \Rightarrow j = 47.244^0$$ below the horizon.