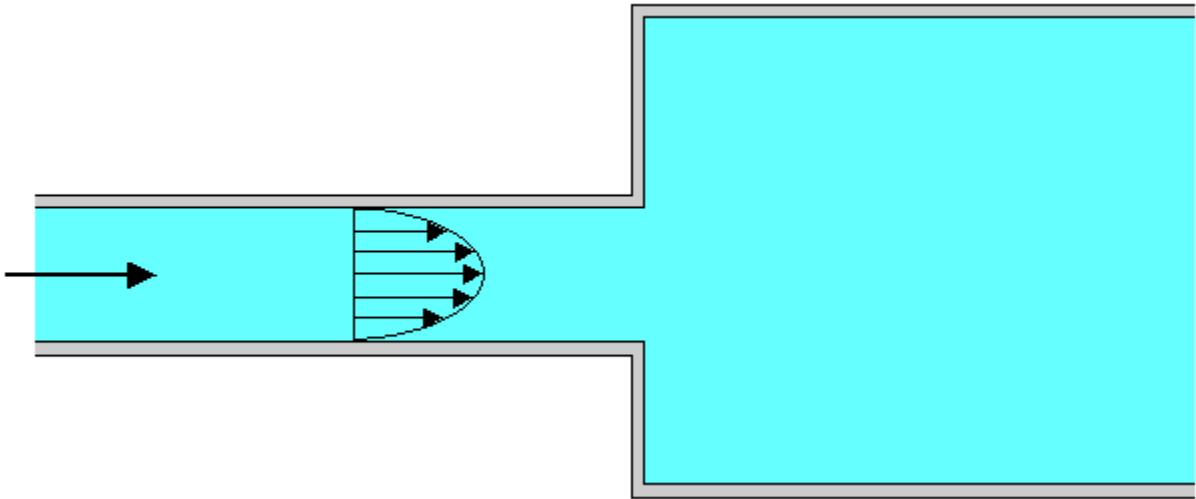


Case 7: Multiple Entries, Compressible, Unsteady Flow with Nonuniform Velocity Profiles at some or all entry points

In all the cases we have considered so far, the velocity we used at each entry / exit point of the control volume represents an average value for the cross-section. In reality, the velocity at each entry point varies across the section according to some (hopefully known) function $V(r)$, assuming a circular cross-section and using cylindrical coordinates. For example, if the flow is laminar, the velocity profile will be parabolic, with its highest value in the middle of the cross-section



$$V(r) = \left[-\frac{d}{ds}(p + \gamma z) \right] \frac{r_0^2 - r^2}{4\mu}$$

where:

s is the direction of the flow (along the centerline of the pipe)

p is the local pressure of the flow

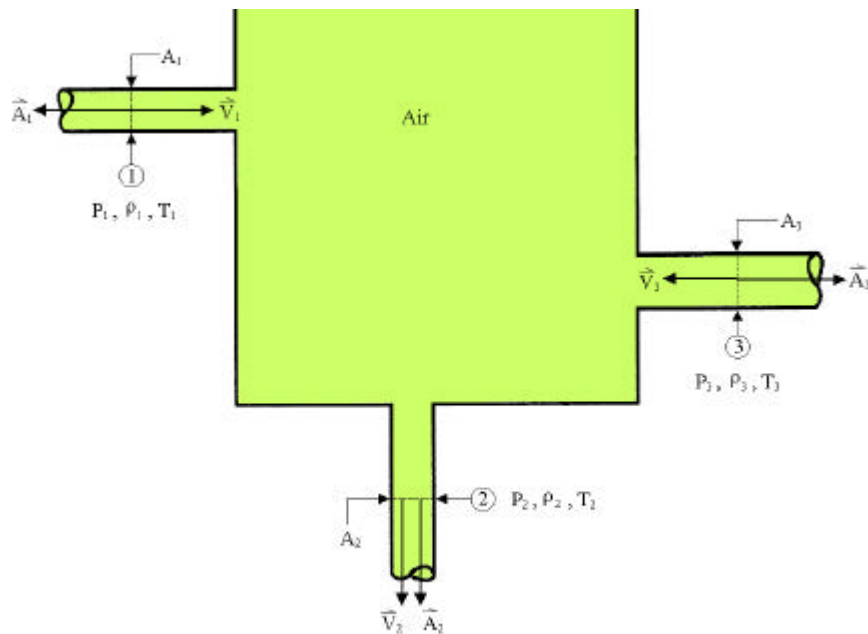
γ is the specific weight of the fluid

z is the local elevation of the fluid

μ is the molecular viscosity of the fluid

r is the local radius measured from the centerline of the pipe.

r_0 is the radius of the pipe itself.



Now consider case 6 but allow for the possibility of nonuniform velocity profiles at each entry / exit point, which is the case in all engineering applications. The mass flow rate at each entry / exit point must now be written as:

$$\dot{m} = \int_{C.S.} \mathbf{r} \cdot \vec{V} \cdot d\vec{A}$$

The unsteady term can still be expressed as in case 6. Thus, the most general form of continuity can be expressed as:

Continuity eq. for unsteady, compressible, 1-D (or quasi 1-D) flow for a control volume with multiple entries and nonuniform velocity profiles at some or all entry points.

$$\int_{C.S.} \mathbf{r} \cdot \vec{V} \cdot d\vec{A} = - \frac{d}{dt} \int_{C.V.} \mathbf{r} \cdot d\forall$$