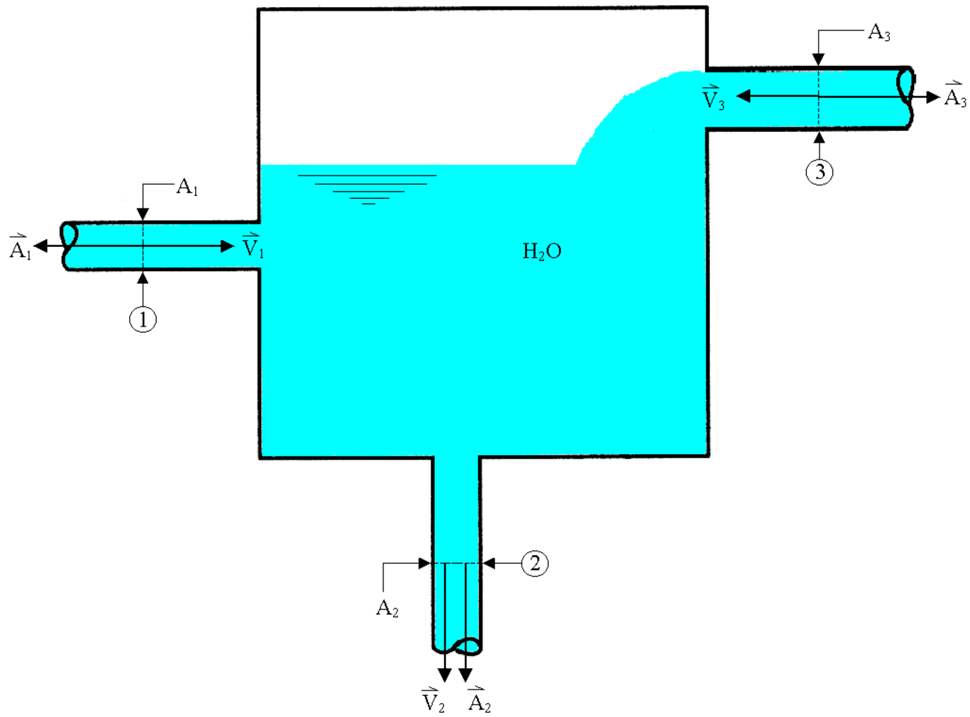
CONTINUITY

<u>Case 5: **Multiple Entries, Incompressible, Unsteady Flow**</u>



Now consider case 4 but assume that the water level in the reservoir does not remain constant. This is an example of incompressible, unsteady flow, since the mass of the water in the tank is changing. It is now possible to have an imbalance between the inflow and the outflow and the difference will show up as either an increase or a decrease in the mass inside the tank. Continuity in this case must be written as:

$$\dot{m}_{in} - \dot{m}_{out} = \frac{d(m_{cv})}{dt} \qquad (1)$$

Eliminating the density (since the flow is incompressible), we get:

$$Q_{in} - Q_{out} = \frac{d(\forall_{H_2O,cv})}{dt} \qquad (2)$$

The left-hand side of eq.(2) can be written as before (case 4), while the time rate of change of the water volume on the right-hand side is simply the product of the cross-sectional area of the tank times the velocity of the surface.

$$-V_1A_1 + V_2A_2 - V_3A_3 = A_{\tan k}\frac{d(depth)}{dt} = A_{\tan k}V_{surface} \qquad (3)$$

Generalizing:

Continuity eq. for **unsteady**, **incompressible, 1-D** (or **quasi 1-D**) flow for a control volume with **multiple entries**

$$\sum_{C.S.}\vec{V}\cdot\vec{A} = -\frac{d\left(\forall_{H_2O,cv}\right)}{dt} \qquad (4)$$

The minus sign on the left side of eq.(3) is necessary because of the convention discussed earlier. The summation on the left side will give a positive result if the outflow is greater than the inflow, in which case the mass inside the control volume is decreasing, so the derivative on the right-hand side will be negative.