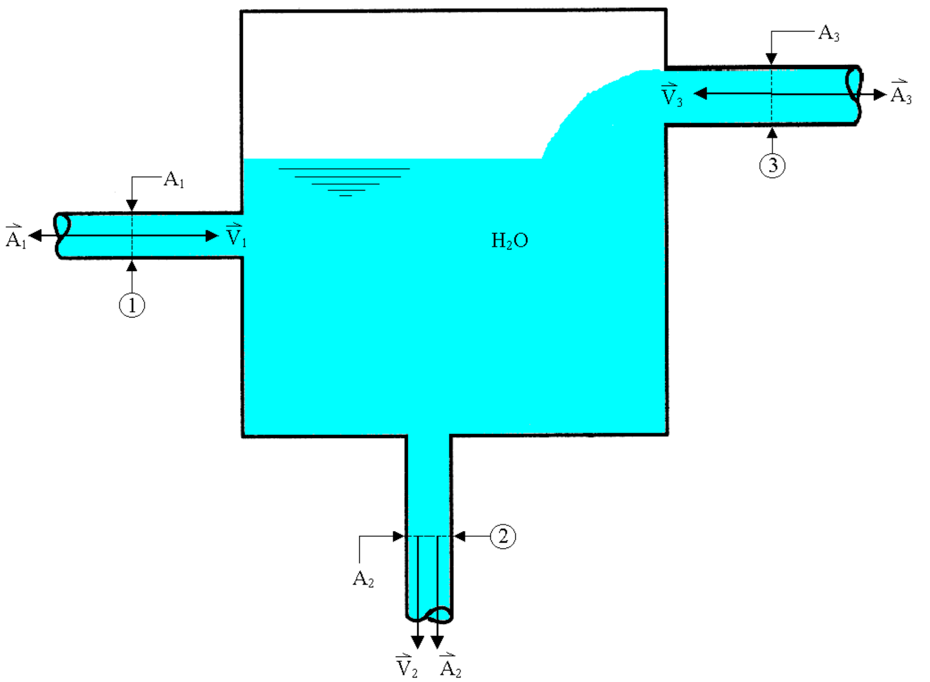
CONTINUITY

<u>Case 4: **Multiple Entries, Incompressible, Steady Flow**</u>



Now imagine we have a water reservoir as shown in the figure. There are two entry points (1 and 3) where water is being fed into the reservoir, and one exit point where water is being drained from the reservoir (point 2). Assume that the water level in the reservoir remains constant, which implies that the mass of the water inside the reservoir does not change (in other words, the flow is steady). Continuity in this case can be written as:

$$\dot{m}_{in} = \dot{m}_{out} \Rightarrow (rQ)_{in} = (rQ)_{out} \qquad (1)$$

and since the flow is incompressible, ρ is the same everywhere so eq.(1) becomes:

$$Q_{in} = Q_{out} \Rightarrow V_1 A_1 + V_3 A_3 = V_2 A_2 \Rightarrow -V_1 A_1 + V_2 A_2 - V_3 A_3 = 0 \qquad (2)$$

Eq.(2) can also be written in vector form:

Continuity eq. for **steady**, **incompressible, 1-D** (or **quasi 1-D**) flow for a control volume with **multiple entries**

$$\sum_{C.S.} \vec{V} \cdot \vec{A} = 0$$

where $\vec{A}$ is the area vector for the cross-sectional area of each entry / exit point of the control surface. $\vec{A}$ is defined perpendicular to the surface it represents, always pointing outwards from the control surface, and its magnitude is equal to the magnitude of the cross-sectional area it represents (in m^2 or ft^2). Thus, the signs for each term in eq.(2) are taken care off by the dot product of the two vectors in the summation symbol. When the flow is going inside the control volume, $\vec{V}$ and $\vec{A}$ point in opposite directions. Therefore, their dot product will give a minus sign from the cosine of 180 degrees, which is the angle between the two vectors. When the flow is leaving the control volume, the two vectors are pointing in the same direction so the angle between them is 0 degrees, the cosine of which is +1. In other words, mass (or in this case volume) rate of flow going in is taken as negative and mass rate of flow going out is taken as positive.