AEROSTATICS

Example Problem: Buoyancy / Balloon

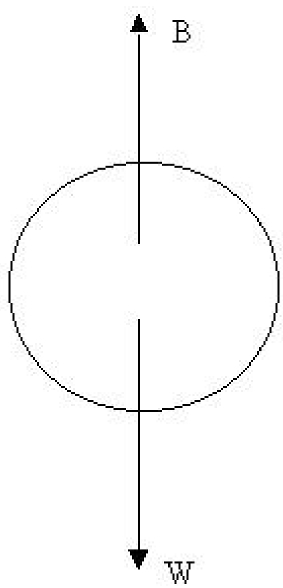
Determine the diameter of a spherical balloon, which will be used to carry meteorological instruments to high altitude.

Given:

Altitude to which balloon will fly:	5 km
Weight of instruments:	25 N
Gas to be used to fill the balloon:	Helium
Weight of material out of which balloon will be made:	0.03 N/m^2

Solution:

From the free-body diagram of the balloon:



$$B = W = W_{instr.} + W_{balloon} + W_{Helium} \quad (1)$$

The buoyancy force can be calculated from Archimedes' law:

$$B = g_{air} \forall_{balloon} = r_{air}\, g \left[\frac{1}{6} p\, D^3\right] \quad (2)$$

where D is the unknown diameter of the balloon. The density of the air depends on the altitude and the temperature. Assuming a standard atmosphere:

$$T = T_{SL} - a\, h = (273 + 15) - 0.0065(5{,}000) = 255.5\, K$$

where $\alpha = 0.0065$ K/m is the lapse rate (indicates how quickly the temperature drops with altitude) and 15^0C is the standard temperature at sea level. From standard atmosphere tables we get:

$$p_{5kn} = 54.008\,kPa$$

Now we can calculate the density of the air and the density of the helium from the ideal gas law:

$$r_{air} = \frac{p}{R_{air}T} = \frac{54,008}{287(255.5)} = 0.74\,kg/m^3 \qquad (3)$$

$$r_{He} = \frac{p}{R_{He}T} = \frac{54,008}{2,077(255.5)} = 0.1\,kg/m^3 \qquad (4)$$

Now we can calculate the weights on the right hand side of eq.(1):

$$W_{He} = g_{He}\forall_{balloon} = r_{He}g\left(\frac{1}{6}p\,D^3\right) \qquad (5)$$

$$W_{balloon} = b_{material}S_{balloon} = 0.03p\,D^2 \qquad (6)$$

Substituting eqs.(2-6) into eq.(1) we get a cubic for D:

$$D^3 - 0.03D^2 - 7.6 = 0 \Rightarrow D = 1.97\,m$$