PING-PONG ANYONE?

Solving a problem in 60 sec (individually)

Look around the room you are sitting in. Take just 60 sec to answer the following question:

How many ping-pong balls could you fit into the room?

• Nothing wrong with just saying “lots” or “thousands”. Considering how much time you’ve had, these answers are definitely better than nothing!

• What type of model did you use?
- Yes, you did use a model! You modeled in terms of categories such as few, some, lots or tens, hundreds, thousands. By drawing a mental picture of a ping-pong ball and looking around the room, you then estimated that the answer belonged in one category rather than another. This is NOT a useless exercise; it makes a difference whether the answer is “some” or “lots”!

• You might have tried to make a rough calculation of the # of ping-pong balls. For example, you might have estimated the volume of a ping-pong ball and the volume of a room, and divided one into another. If you made a volumetric calculation:

What was your model of a ping-pong ball? What was your model of the room?

Did you model the ping-pong ball as a cube rather than a sphere?
Did you model the room as a large, empty box?

What other simplifications or assumptions did you make?

Did you ignore the furniture?
Did you assume that you were not allowed to squash or deform the balls?

Solving the same problem in 5 min (with a partner)

• Keep a note of how you go about solving it.

• How did you go about solving the problem this time?
Did you use the same procedure but refine your measurements?

Or did you use the extra time to take a new approach?
Did you change your model?
Did you modify your assumptions? Did you, for example make a correction for the furniture or the shape of the room?

• What were the differences between working alone and working in a team?
Did you share tasks?
Did you start with similar ideas or did you spend time arguing about the proper approach?

• Given more resources (time and people), chances are you built a more sophisticated model. For example:

$$V_{room} = LWH$$

$$V_{ball} = D^3$$

$$\# \text{ of balls} = V_{room} / V_{ball} = LWH / D^3$$

<u>What have we learned about modeling?</u>

Take 10 min to make a list of the points that you think these two exercises illustrate. This is NOT an easy task!

1. A model is a partial rather than a complete representation.

2. Even a very rough answer is better than no answer at all.

3. A model that is inadequate under one set of circumstances may be the best you can come up with under another set of circumstances. In other words, the design of a model depends as much on circumstances and constraints (of money, time, data, and personnel) as it does on the problem that is being solved. It follows that the assumptions one makes depend on the circumstances in which one solves the problem.

4. A symbolic representation (i.e., choosing a notation and building a formula or formulae) is clean and powerful. It communicates, simply and clearly, what the modeler thinks is important, what information is needed, and how that information will be used.

5. Sometimes one uses models implicitly (i.e., without being aware of doing so). At other times one consciously and explicitly constructs or uses a model. An explicit model is an indispensable tool for solving problems and talking about the solution.

The Real World vs. the Model

• A model is like a caricature. It picks on certain features (a nose, a smile, or a lock of hair) and concentrates on them at the expense of the other features. A good caricature is one in which these special features have chosen purposefully and effectively.

In the same way a model concentrates on certain features of the real world. Whenever you build a model you have to be selective. You have to identify those aspects of the real world that are relevant and ignore the rest. You have to create a stripped down model world which enables you to focus, single-mindedly, on the problem you are trying to solve.

Metaphor: Occam's razor
Occam: 14th century English philosopher: "Things should not be multiplied w/o good reason" - i.e.,
eliminate all unnecessary information
relating to the problem you are analyzing

Why Models are important

One needs to know not only your answer to the problem (for ex. 28,517 balls) but also how you got there so that he/she can evaluate your answer. If you are not aware of the model you are using you will not be able to communicate your answer in a meaningful way.

What was the first question you asked your partners:
- What did you do in the 60 sec exercise? or
- How many balls did you come up with?

If you had more time...

• What is the best answer you could give?
Do you recommend measuring the room and the ball accurately? Or,
Fill the room up with ping-pong balls and count them!
Can you think of a better answer than that?
Is it worth the effort?
How good of an answer are you willing to accept?

Can't answer any of these questions unless we know first:
Why on earth do we want to fill this room with ping-pong balls to begin with?

Upper and Lower Bounds

Is the answer you got from the eq. derived earlier too small or too large? In other words are you underestimating or overestimating the # of balls in the room?

Treating the ball as a cube will give you a lower bound to the answer.

If you model the ping-pong ball as a sphere instead of a cube, then:

$$V_{ball} = (4\pi/3)(R)^3$$

Now imagine the spheres packed into the room so that there are absolutely no air gaps between them. Then:

$$\text{\# of balls} = V_{room} / V_{ball} = LWH / (4\pi/3)(R)^3$$

This eq. will give you an upper bound to the answer.

If you divide the two answers you get

$$\text{upper bound / lower bound} = (6 / \pi) = 2 \text{ approximately}$$

In other words, your model is about 50% accurate, simply because you did not consider how the balls will be packed together!

• Ranking the assumptions you made. Which one(s) should you relax first?
• Every time we relax an assumption, more data (measurements?) and new assumptions will be needed! Eventually we have to ask the question: what is the appropriate resolution for our model?

Reference: A.M.Starfield, K.A. Smith, A.L. Bleloch: "How to Model it; Problem Solving for the Computer Age", McGraw-Hill, 1990.