Inertia Calculations

Cylindrical object (solid) of diameter D (radius R=D/2) and mass, M

\[ J_{aa} = \frac{1}{2} MR^2 = \frac{1}{8} MD^2 \]
\[ = \frac{1}{2} \pi \rho l R^4 = \frac{1}{32} \pi \rho l D^4 \]

\( J_{aa} \) = mass moment of inertia about axis aa (polar moment of inertia)

\( \rho \) = density of the material

\( l \) = length of cylinder

Cylindrical object (hollow) with inner diameter \( D_i \) (radius \( R_i = D_i/2 \)), outer diameter \( D_o \) (radius \( R_o = D_o/2 \)), and mass, M

\[ J_{aa} = \frac{1}{2} M(R_o^2 + R_i^2) = \frac{1}{8} M(D_o^2 + D_i^2) \]
\[ = \frac{1}{2} \pi \rho l (R_o^4 - R_i^4) = \frac{1}{32} \pi \rho l (D_o^4 - D_i^4) \]

Direct drive load

\[ J_{tot} = J_{motor \ armature} + J_{load} \]

*Note: Shafts do have inertia, but their contribution to \( J_{tot} \) is often negligible. Why?
Gear driven load

\[ J_{\text{tot}} = J_{\text{motor armature}} + J_{\text{gear 1}} + (N_1/N_2)^2 \left[ J_{\text{gear 2}} + J_{\text{gear 3}} + (N_3/N_4)^2 \left\{ J_{\text{gear 4}} + J_{\text{load}} \right\} \right] \]

\( N_i \) is the number of gear teeth on gear \( i \). \( N_i/N_j \) is the gear ratio between gears \( i \) and \( j \).

(Note that the polar moment of inertia terms in the equation above refer to their central principal values about their axes of rotation)

Leadscrew driven load

\[ J_{\text{tot}} = J_{\text{motor armature}} + J_{\text{leadscrew}} + \frac{M}{(2\pi p)^2} \frac{1}{e} \]

\( p \) = leadscrew pitch (threads/length)
\( e \) = efficiency of leadscrew
\( M \) = mass of load
\( \rho \) = density of leadscrew material

Tangentially driven load

\[ J_{\text{tot}} = J_{\text{motor}} + J_{\text{pulley 1}} + J_{\text{pulley 2}} + MR^2 + M_{\text{belt}}R^2 \]
where \( J_{\text{pulley i}} \) is the polar moment of inertia for pulley \( i \) about its rotational axis, \( M_{\text{belt}} \) is the mass of the belt, and \( R \) is the radius of both pulleys.